FAR-Wake
Fundamental Research on Aircraft Wake Phenomena

Specific Targeted Research Project
Start: 01 February 2005
Duration: 40 months

Numerical investigations of end-effects associated with accelerated/decelerated wings: time-developing and space-developing simulations

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Document control data

<table>
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<th>Deliverable No.</th>
<th>TR 1.1.2 - 6</th>
<th>Due date:</th>
<th>November 2007 (m32)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Version:</td>
<td>1.0</td>
<td>Task manager:</td>
<td>S. Le Dizès</td>
</tr>
<tr>
<td>Date delivered:</td>
<td>13 February 2008</td>
<td>Project manager:</td>
<td>T. Leweke</td>
</tr>
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<td>EC Officer:</td>
<td>R. Dénos</td>
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Project co-funded by the European Commission within the Sixth Framework Programme (2002-2006)

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<td>CO Confidential, only for members of the consortium (including the Commission Services)</td>
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TR1.1.2-6 “Numerical investigations of end-effects associated with accelerated/decelerated wings : time-developing and space-developing simulations”

T. Lonfils, R. Cocle, G. Daeninck, C Cottin and G. Winckelmans

September 28, 2007

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Abstract

In this report, we investigate the end-effect phenomenon occurring due to the variation of the circulation in a vortex wake. We first focus on comparing a numerical simulation to an experiment made by CNRS/IRPHE[11]. Two types of instabilities occur: an axisymmetric one produced by the circulation variation, and generating an axial velocity which can lead to a helical one, as it is presently the case, if this velocity is strong enough. We also compare some results to the theory derived for the Lamb-Oseen vortex. We then investigate a more realistic case of end-effects associated with an accelerated/decelerated wing. Analysing the corresponding time-developing and space-developing (S-D) simulations, an explanation of the different behavior in the accelerated and in the decelerated cases is proposed. A characterization of the net result of the wake rollup, in terms of tangential velocity profile, axial velocity, and their evolution in the wake, was also done, using the results of the S-D simulation. The fits of these profiles with the most usual vortex models was also done. In particular, we show that the two length-scale models of Proctor-Winckelmans and of Jacquin provide a very good fit (both in terms of maximum tangential velocity and effective core size).

1 Introduction

Due to the aircraft lift, wake vortices are inevitably generated. They can be hazardous for a follower aircraft and are then an important issue to ensure a high security level in air traffic. The purpose of this work is to characterize the typical vortex instabilities occurring when aircraft accelerate/decelerate, which could lead to the disappearance of the vortex coherence: this is why such instabilities are of interest. Theoretical and numerical investigations have already been carried out, but they remain academic as they are applied to the Lamb-Oseen vortex (see [10, 5, 12, 4, 9]).

In the first part of this report, we perform a numerical simulation to be compared to an experiment also carried in the framework of FAR-Wake [11] in which the axial propagation of vortex perturbations was investigated.

Two other simulations are also carried out. The first one is time-developing, and thus assumes periodicity. The second one simulates the complete spatial evolution of the wake, which evolves using an inflow condition. The latter constitutes a better representation of the reality, but it requires much more computational resources. Both cases are analyzed and compared.

Experimental investigations of end-effect generated by a accelerated/decelerated wing have also been carried out using the UCL/TERM facilities ([8], [7]). The influence of the lift coefficient, the flight speed and the acceleration/deceleration intensity have also been studied. They confirm that the propagation speed is of the order the maximal tangential velocity. Fig. 1 shows a helical instability propagation for a deceleration case.

Because vortex lines have to be closed, the increase of the wake circulation is associated with the creation of spanwise vorticity component. The general description of the corresponding wake configuration is depicted in Fig. 2. The global dynamics are essentially dependent on the ratio of the net increase of circulation (i.e. \( \Delta \Gamma = (\Gamma_1 - \Gamma_0)/\Gamma_1 \)) over the acceleration distance \( E/b_0 \) ([8]).

The so-called “end-effect” phenomenon (relative to a sudden acceleration/deceleration of the wing) corresponds to the particular case with \( \Gamma_0 = 0 \); but the term will here be also used to qualify the general case (i.e. \( \Gamma_0 \neq 0 \)).
**Figure 1:** Dye visualization of instabilities generated by a decelerated wing. The fluorescein is injected at the wingtips (for more details, see [8]).
2 Numerical method

For the simulations, we use the Vortex-in-Cell method combined with the Parallel Fast Multipole Method (called VIC-PFM, for short). It is based on a combination of Lagrangian and finite difference methodologies, and it solves the vorticity form of the incompressible Navier-Stokes equations

\[
\frac{D \omega}{Dt} = \nabla \cdot (u \omega) + \nabla \cdot W, \tag{1}
\]

in which \(D/Dt = \partial/\partial t + (u \cdot \nabla)\). The divergence of the diffusive vorticity flux tensor \(W\) is written here

\[
\nabla \cdot W = \nu \nabla^2 \omega + \nabla \cdot \left( \nu_{sgs} \left( \nabla \omega + (\nabla \omega^s)^T \right) \right), \tag{2}
\]

where \(\nu_{sgs}\) is the effective subgrid-scale viscosity and \(\omega^s\) is the “small-scale” part of the LES field (since we here use the multiscale subgrid-scale approach).

The numerical solution of Equation (1) is sought in a two-fold approach. First, the discrete vorticity field is constructed from a sum of regularized vortex particles of strength \(\alpha\) such that

\[
\omega(x, t) = \sum_p \alpha_p(t) \frac{1}{(\sqrt{2\pi} \sigma_p)^3} \exp \left( -\frac{1}{2} \frac{|x - x_p(t)|^2}{\sigma_p^2} \right), \tag{3}
\]

in which \(\alpha_p = \int \omega \, dx = \omega_p h^3\), \(x_p\) is the particle position and \(\sigma_p\) its regularization parameter [3, 14]. The convective part, or lhs of Equation (1), is evaluated using a Lagrangian approach with \(d x_p/dt = u(x_p)\). For the second part, the time variation of the vortex particle strength, i.e., the rhs of Equation (1) that includes both the vortex stretching and dissipation terms, it is solved on a regular grid using 2nd order finite differences. Interpolation from the Lagrangian particles to the Eulerian grid is done using the \(M_4\) scheme [3, 14]. Once on the Eulerian grid, the stream function vector \(\psi\) is evaluated by solving the Poisson equation

\[
\nabla^2 \psi = -\omega. \tag{4}
\]

The velocity field, needed for convection and stretching, is then obtained by evaluating \(u = \nabla \times \psi\) using again 2nd order finite differences. The global time marching procedure is carried out using a 2nd order leap Frog scheme for the convection and the 2nd order Adams-Bashford scheme for the stretching and dissipation terms. Finally, the divergence-free character of the vorticity vector field is insured by a proper reprojection of the vorticity field (which also requires solving a Poisson equation).

Particular to the present implementation of this procedure is the treatment of the boundary conditions. The idea is here to work on a domain that contains tightly the vorticity field and that can be decomposed in several subdomains on which the exact boundary conditions are obtained using a Parallel Fast Multipole (PFM) method. This amounts solving a 3-D Poisson equation without any iteration between the subdomains (e.g., no Schwarz iteration required): this is so because the PFM method has a global view of the entire vorticity field. So, at any given time step, the boundary conditions for \(\psi\) are obtained using the Green’s function approach via the PFM method. This allows for a significant reduction of the computational cost for solving Equation (4) —as it retains proper open domain conditions and the capability to make parallel subdomain
decomposition simulations—when comparing with more classical VIC methodologies [13, 2].

The LES modeling is here done using a multiscale subgrid-scale model: the model acts on the small scale part of LES field, \( \omega_s \), only. It is obtained from the complete LES field, \( \omega \), using a compact discrete filter (applied iteratively). We here use the Regularized Variational Multiscale (RVM) model [1]. The advantage of such model is that it preserves the inertial range while providing dissipation at the high wave numbers: it is only active during the complex phases of the flow, while remaining inactive during the phases when the vortices are coherent and well-resolved.

3 Numerical simulation of end-effects within vortex pairs: analysis and comparison with the IR-PHE experiments.

3.1 Experimental setup

In this section, we present the numerical investigation carried out in order to compare with one of the CNRS/IRPHE experiments. In their experiments, the vortex is generated in a water tank using a plate which is rotated (for more details, see [11]). We here focused on the case at moderate Reynolds number, i.e. \( Re_\Gamma = 3960 \), in order to be able to afford an equivalent Direct Numerical Simulation (DNS).

3.2 Numerical setup & initial condition

We investigated the case of a vortex pair spaced by \( b_0 \) and linked at the extremities as depicted in Fig. 2 and Fig. 3.

![Figure 2: Schematic of the wake vortex lines corresponding to an accelerated and decelerated wing. Flow periodicity is here assumed over the length \( L_0 + L_1 \).](image)

This case is also quite similar to the wake generated by a wing suddenly accelerated and decelerated (i.e. \( \Gamma_0 = 0 \)), assuming that the vortex pair is immediately formed. The vortices are initialized using the Low Order Algebraic (LOA) vortex model:

\[
\frac{\Gamma(r)}{\Gamma_1} = \frac{r^2}{r^2 + r_c^2}, \quad u_\theta(r) = \frac{\Gamma(r)}{2\pi r},
\] (5)
where $r_c$ is the effective vortex core size (defined as the radius where the tangential velocity $u_\theta(r)$ is maximum). Here we use $r_c = 0.05b_0$. We thus initially have

$$u_{\theta,max} = \frac{\Gamma_1}{4\pi r_c} = \frac{1}{2} \left( \frac{\Gamma_1}{2\pi b_0} \right) \left( \frac{r_c}{b_0} \right) = \frac{1}{2} V_0 \left( \frac{r_c}{b_0} \right) = 10V_0,$$

(6)

where $V_0$ is the wake descent velocity. The Reynolds number, based on the circulation, is here chosen as $Re = \Gamma_1/\nu = 5000$. The distance of the reconnection $E$ and the vortex length $L$ are respectively equal to $1.0b_0$ and $20b_0$. The time step and the mesh size are fixed to $\Delta t = 3.98 \times 10^{-4}$ and $h/b_0 = 1/64$. The simulation was carried out using 60 million grid points. It ran on 16 processors during 240 hours.

### 3.3 Analysis of the results

Since the problem is left/right symmetric and begin/end symmetric, we focus on only one vortex, without loss of generality.

**Evolution of the vortex core size** The instantaneous circulation profile $\Gamma(r)$ is defined by

$$\Gamma(r) = \int_0^{2\pi} \int_0^r \omega_x(r', \theta) r' \, dr' \, d\theta.$$

(7)

It is well known that the end-effect phenomenon much increases the effective vortex core. Moreover, in the current simulation, the Reynolds number is moderate and thus the increase of the vortex core by diffusion is also not negligible. Using the analogy of
the Gaussian vortex diffusion, one characterizes the evolution of the LOA vortex core by diffusion as

\[ r_c^2(t) \simeq r_c^2(0) + \alpha \nu t, \]

where \( \alpha \) is found to be equal to roughly 7.9 (using the current simulation, we analyzed the evolution of the vortex core at the cross plane \( x/b_0 = 10 \) when it is not yet affected by the end-effect) as shown in Fig. 4.(a), instead of 4.0 for the Gaussian vortex. We can also estimate a priori the evolution of the maximum tangential velocity, assuming that the vortex remains close to the LOA model:

\[ u_{\theta,max}^{LOA}(t) = \frac{\Gamma_1}{4\pi r_c(t)}. \]

Fig. 4.(b) shows the evolution of the measured maximum tangential velocity as based on the circulation profile assuming a LOA model. It turns out that the maximum tangential velocity decreases significantly during the simulation, and that both cases are quite different. The case based on the LOA model underestimates the velocity during the whole simulation: this means that the vortex structure doesn’t remain like the LOA one.

Fig. 5 presents the evolution of the vortex core size \( r_c \) for three cross planes: \( x/b_0 = 5.0, x/b_0 = 7.5 \) and \( x/b_0 = 10.0 \). Each shows the same global evolution. At first, the vortex core increases purely by diffusion and then suddenly magnifies when the perturbations cross the plane. We can then calculate the propagation speed of the instability as the ratio of the distance between two cross planes, to the time difference when the vortex core increases:

\[ u_c = \frac{\Delta x}{\Delta t}. \]

Here we find \( u_c/u_{\theta,max}(0) \simeq 2.5/2.8 = 0.89. \)

**Figure 4:** (a) Time evolution of the effective core size \( r_c \) (thick line) and its least-square fitted line (thin line). (b) Time evolution of the maximum tangential velocity \( u_{\theta,max}(t) \) (solid line), and also that assuming that the LOA model remains valid (dashed line).

We also analyze the potential hazard of the vortex. The total circulation of the vortex \( \Gamma_1 \) is not a good estimation since it is conserved. We use \( \Gamma_{5-15} \) : the mean of the vortex
Figure 5: Evolution of the vortex core size $r_c$ for various cross plane: $x/b_0 = 2.5$ (dash-dotted line), $x/b_0 = 5.0$ (dashed), $x/b_0 = 7.5$ (dotted line) and $x/b_0 = 10.0$ (solid line).

Figure 6: Circulation $\Gamma_{5-15}$ as a function of time for the cross plane $x/b_0 = 5.0$.

circulation between 5 and 15 meter for an aircraft with a 60 meter wingspan. It reads

$$\Gamma_{5-15} = \frac{1}{b/6} \int_{b/12}^{b/4} \Gamma(r) \, dr,$$  \hspace{1cm} (10)

where $b$ is here taken as $b = (4/\pi)b_0$ (assuming a wing with elliptical loading). Fig. 6 shows its evolution: we clearly observe a net diminution at $tu_{\theta,max}/b_0 \simeq 4.8$, due to the passage of the perturbations.
Propagation of the perturbations There are several manners to measure the speed of the perturbation. The conclusion of the observations of Meunier in its experiment[11] is that there would be two main instabilities. The first one is supposed to be a “front wave” with a strong axial velocity within the vortex core (as shown in Fig. 7.a); the second would be a helical instability which strongly deforms the vortex (as shown in Fig. 7.b) and would propagate at a lower speed.

Figure 7: (a) Longitudinal section of the vortex at \( y = b_0/2 \). Axial component of the velocity field and front velocity the non-zero axial velocity \( (u_w) \). (b) Side view of the vorticity norm isocontours ||\( \omega || \) and description of the isophase point speed \( (u_{ip}) \) and its front speed \( (u_{if}) \).

Fig. 8 shows the time evolution of the front position of the non-zero axial velocity. For the adimensionalisation, we used the characteristic length scale \( b_0 \) and the maximum tangential velocity as the velocity scale. We compared the propagation speed using three different tangential velocity : (a) the time evolving maximum tangential velocity \( u_{\theta,\text{max}}(t) \) and (b) the initial maximum tangential velocity \( u_{\theta,\text{max}}(0) \). As foreseen, the propagation speed is of the order of the maximal tangential velocity in each case : 1.5 and 0.86 respectively. The viscous diffusion of the vortex core here plays a significant role in slowing down the propagation, as the tangential velocity also diminishes. We can also compare the propagation speed to the theoretical one predicting the propagation of the group velocity of an axisymmetric mode for a Gaussian vortex (see Fabre et al.[4] and Jacquin et al.[10]) :

\[
u_w \simeq 0.63 \frac{\Gamma_1}{2\pi\sigma} \tag{11}\]

where \( \sigma \) is the Gaussian vortex core siz. To compare to that, we here consider the Gaussian vortex which has the same long wavelength dynamics as the LOA vortex : this give \( \sigma_{eq} = 1.34 r_c \). Here we find \( u_w/(\frac{\Gamma_1}{2\pi\sigma_{eq}}) \simeq 0.57 \) indeed close to the theoretical value. In the experiment, the wave perturbation speed was above : roughly 0.9.

Another important information is the swirl parameter \( S \) which is the ratio between the maximum tangential velocity and the maximum axial velocity in the vortex. Here we
find $S$ being nearly equal to $0.9 \ldots 1.2$. A value included in the interval $[0, 1.5]$ means that the helical Kelvin mode of the vortex is unstable.

![Figure 8](image_url)

**Figure 8:** Position of the non-zero axial velocity front (solid) and its least-squared fitted line (dashed) for various time scaling based on: (a) the time evolving maximum tangential velocity $u_{\theta,\text{max}}(t)$, (b) the initial maximum tangential velocity $u_{\theta,\text{max}}(0)$.

We then analyze the propagation of this helical perturbation. The time evolution of the isophase points (as described in Fig. 7.b with the red lines) and the position of their front are presented in Fig. 9. We find a group velocity equal to $u_{gf}/u_{\theta,\text{max}}(0) = 0.85$. This velocity is the same as the wave speed $u_w$ (which creates the axial velocity). The reason could be the fact that the axial velocity generated by the wave is strong enough to permit the helical instability to immediately occur. This is the main difference with the experimental results for which the wave speed was twice the speed of the isophase front. Meunier suggests that the speed of the isophase point compares with the phase velocity of the helical Kelvin mode ($m = 1$). We find in our case a perturbation wavelength of $\lambda/r_c = 16$ (taken where the instability is inside the vortex core) and thus a wavenumber $k r_c = 0.39$. The theoretical phase velocity of the helical mode of a Lamb-Oseen vortex with this wavenumber is

$$0.33 \frac{\Gamma_1}{2\pi \sigma}. \quad (12)$$

In our case we find $u_{ip} \simeq 0.23 \Gamma_1/(2\pi \sigma_{eq})$.

**Visualization of the perturbation** We define the vorticity perturbation field as

$$\omega = \omega^{(0)} + \omega', \quad (13)$$

where $\omega^{(0)}$ is the “base” field (i.e. the vorticity field that would exist without the perturbation). Here, $\omega^{(0)}$ is taken as the field at the center slice, i.e. at $x/b_0 = 10$. $\omega'$ is thus the vorticity perturbation field. Isocontours of $\omega_x'$ for two different times are presented in Fig. 10.

Fig. 11 shows the comparison between the numerical simulation and the experiment for the axial vorticity component at a fixed slice for various times. The global dynamics...
is roughly the same as in the experiment. The first phase consists in the simple laminar diffusion of the vortex. The effective vortex core then grows suddenly when the wave crosses the slice, with loss of coherence after that.

3-D view of vorticity norm isocontours are also presented in Fig. 12, clearly slowing the helical instabilities.

Figure 9: Evolution of the deformation front (dash) and of the isophasic points (dash-dot). The solid lines correspond to the least-squared fitted lines.

Figure 10: Isocontours of the vorticity perturbation axial component $\omega'_x b_0^2/\Gamma_1$ at $x/b_0 = 5.0$ at $tu_{\theta,max}/b_0 = 6.0$ (left) and 12 (right): ten equally spaced levels from $-4.5$ to 4.5; positive levels are in red and negative levels are in blue. The vortex center and the effective core radius $r_c$ are also shown.
Figure 11: Visualization of the vorticity field axial component for the cross plane $x/b_0 = 5.0$. Numerical simulation (left) and IRPHE experiment (right). The size of the visualization boxes are $15.5r_c$ and $11.1r_c$ respectively. The time between two plots is $\Delta t u_{\theta,max}/r_c \simeq 24$. For the numerical simulation, a circle using a radius $r_c(t)$ centered on the vorticity centroid is also plotted.
$t u_{\theta, \max}(0)/b_0 = 0.0 \left( t \frac{\Gamma_1}{2 \pi b_0} = 0 \right)$

$\theta = \max(0)/b_0 = 1.6 \left( t \frac{\Gamma_1}{2 \pi b_0} = 0.16 \right)$

$\theta = \max(0)/b_0 = 3.2 \left( t \frac{\Gamma_1}{2 \pi b_0} = 0.32 \right)$

**Figure 12:** Visualization of the vorticity norm isosurfaces for $||\omega||b_0^2/\Gamma_1 = 1.0$ (low opacity), 5.0 (medium opacity) and 10 (high opacity), colored with the axial vorticity component.
\[ tu_{\theta,\text{max}}/b_0 = 4.8 \left( t \frac{\Gamma_1}{2\pi b_0} = 0.48 \right) \]

\[ tu_{\theta,\text{max}}/b_0 = 6.4 \left( t \frac{\Gamma_1}{2\pi b_0} = 0.64 \right) \]

\[ tu_{\theta,\text{max}}/b_0 = 8.0 \left( t \frac{\Gamma_1}{2\pi b_0} = 0.080 \right) \]

**Figure 12:** [Cont’d]
\[ t_{u_{\theta,\text{max}}} / b_0 = 9.5 \left( t \frac{\Gamma_1}{2 \pi b_0} = 0.95 \right) \]

\[ t_{u_{\theta,\text{max}}} / b_0 = 11 \left( t \frac{\Gamma_1}{2 \pi b_0} = 0.11 \right) \]

\[ t_{u_{\theta,\text{max}}} / b_0 = 13 \left( t \frac{\Gamma_1}{2 \pi b_0} = 0.13 \right) \]

**Figure 12: [Cont’d]**
4 Time-developing and space-developing simulations of an accelerated/decelerated wing.

We now investigate the phenomenon of the acceleration/deceleration of a wing. The behavior of the flow over the wing will not be captured by the simulations but modeled using the lifting line approach. This theory (i) neglects the effects of the boundary layers and (ii) “replaces” the wing by a vortex line which has a certain circulation profile $\Gamma(y)$ along the spanwise direction. The lift profile and the circulation profile are related by $l(y) = \rho U \Gamma(y)$ (where $U$ is the flight speed and $\rho$ the fluid density.). Since the vorticity field is divergence free, a vortex sheet of circulation per unit length $\gamma(y)$ has to be shed such that

$$\gamma(y) = \frac{d\Gamma}{dy}(y)$$ \hspace{1cm} (14)

We can then express the total lift of the wing as

$$L = \int_{-b/2}^{b/2} l(y) \, dy = \rho U \int_{-b/2}^{b/2} \Gamma(y) \, dy,$$ \hspace{1cm} (15)

We define $\Gamma_0$ and $b_0$, respectively the total circulation of the half plane and the spacing between the vorticity centroids (and thus also the vortex spacing after rollup). Then

$$\int_{-b/2}^{b/2} \Gamma(y) \, dy = \Gamma_0 b_0$$ \hspace{1cm} (16)

and

$$L = \rho U \Gamma_0 b_0.$$ \hspace{1cm} (17)

Let’s now consider an elliptic wing which has the following characteristics:

- wingspan $b$
- Surface $S$
- Aspect ratio $A_r \triangleq b^2/S$

The lift can also be related to the aerodynamic characteristic of the wing $C_L \triangleq L/\left(\frac{1}{2}\rho U^2 S\right)$. We can thus express the characteristics of the wake ($\Gamma_0$ and $b_0$) as a function of the wing characteristics:

$$\Gamma_0 b_0 = \frac{1}{2} U S C_L = \frac{1}{2} U b^2 \frac{C_L}{A_r}$$ \hspace{1cm} (18)

Here we use

$$C_L = 1.5, \quad A_R = 7.5, \quad \text{thus} \frac{C_L}{A_R} = 0.20, \quad Re = \Gamma_0/\nu = 10^4.$$

The generated vortex sheet rolls up and eventually forms two counter-rotating wake vortices. In the previous section the initial condition was two vortex tubes reconnected at the extremities. We don’t do it anymore. Indeed, we have to compare the relative characteristic time of each phenomenon to be sure that the time/space-evolution of the wake is properly investigated. The characteristic time of the vortex wake $t_0 = b_0/V_0$ has
to be compared to the characteristic time of the perturbation propagation $t_p = b_0/V_p$. Therefore,

$$t_0/t_p = \frac{V_p}{V_0} \approx \frac{u_{\theta,\text{max}}}{V_0}$$

where $V_p$ is the propagation speed of the perturbation which is of the order of the maximum tangential velocity $u_{\theta,\text{max}}$ of the vortex; a typical value is $u_{\theta,\text{max}}/V_0 \approx 10$. It means that we have to take into account the phenomenon of rollup.

Moreover, the third phenomenon is the flight itself which has a characteristic time $t_f = \frac{V_0}{U}$. The ratio of the latter characteristic $t_f$ and $t_p$ is

$$t_f/t_p = \frac{V_p}{U} = \frac{V_p}{V_0} \frac{V_0}{U} \approx \frac{u_{\theta,\text{max}}}{V_0} \frac{4}{\pi^3} A_r \approx 0.2 \ldots 0.3$$

It means that, a priori, the end-effect of the vortex will clearly act in the wake region and won’t be affected by the space-developing evolution of the wake. However, this “global” analysis neglected the influence of an axial velocity in the vortex core region due to the 3-D space-developing rollup (see [12] and [5]). Hence, we also investigate the space-developing case.

So, we propose to investigate and to compare the two cases: (i) the space-developing (S-D) simulation and (ii) the time-developing (T-D) simulation. Fig. 13 presents the comparison between those two concepts.

(i) The S-D simulation consists in three phases. We first simulate the complete space evolving rollup of the vortex sheet shed by a wing flying at a constant speed $U_0$. This is the initial condition. During the second phase, the wing accelerates to the speed $U_1$. And, finally at the time of the third phase, the wing flies at the constant speed $U_1$. The instability evolves timely and is also convected in the wake moving towards the higher circulation region. Since there are an inflow and an outflow, the period of validity, and thus of analysis, is limited by the time needed to convect the instability out of the simulation domain.

(ii) The T-D simulation assumes that the whole space evolution of the wake, including the three phases, is initially generated. We then use periodic boundary conditions in the flight direction. The initial condition is thus a vortex sheet over a distance $L_0$ (corresponding to a wing flying at a speed $U_0$), an acceleration to the speed $U_1$ over a distance $E$, and another piece of vortex sheet over the distance $L_1$ corresponding to the one of the flight speed $U_1$. To match with the periodic boundary conditions, we also add a deceleration phase (also over distance a $E$).

The vortex sheet generated by the wing has to be regularized in order to produce a regular vorticity field to be provided to the VIC-PFM code. For that, we use convolution with a Gaussian:

$$\omega_x(y, z) = \frac{1}{\pi \sigma^2} \int_{-b/2}^{b/2} \exp \left( -\frac{(y - y')^2 + z^2}{\sigma^2} \right) \gamma(y')dy', \quad (19)$$
$t \frac{\Gamma_1}{2\pi b_0} = 0.0$

S-D :

T-D :

$L_x$

$x_{ref}$

$L_0$

$L_1$

$E$

$\hat{e}_x$

$\hat{e}_y$

$\hat{e}_z$

$t \frac{\Gamma_1}{2\pi b_0} = 0.052$

S-D :

T-D :

$t \frac{\Gamma_1}{2\pi b_0} = 0.34$

S-D :

T-D :

$t \frac{\Gamma_1}{2\pi b_0} = 0.68$

S-D :

T-D :

**Figure 13:** Comparison of time-developing (T-D) and space-developing (S-D) simulations: top view of the vorticity field at various times. The zone in gray corresponds to an example of zoom boxes, moving in the S-D case and fixed in the T-D case, used for the 3-D visualizations in Fig. 24, 25 and Fig. 26.
where $\sigma$ is the regularization parameter of the vortex sheet. For an elliptic wing, the span loading (i.e. circulation profile) also has an elliptic profile

$$
\Gamma(y) = \frac{2 UC_L b}{\pi A_r} \left( 1 - \left( \frac{y}{b/2} \right)^2 \right)^{1/2}.
$$

Notice that $\gamma(y)$ becomes infinite at $y = \pm b/2$, but not $\Gamma(y)$. To avoid numerical problems, one rewrites Eq.19 using integration by part:

$$
\omega_x(y, z) = \frac{2}{\pi \sigma^4} \int_{b/2}^{-b/2} \exp \left( -\left( \frac{y - y'}{\sigma^2} \right)^2 + z^2 \right) (y - y')\Gamma(y')dy'.
$$

\section{4.1 Numerical setup & initial condition for the space-developing simulation}

We investigate the S-D behaviour of the wing acceleration/deceleration with a constant lift coefficient $C_L$. It means that the lift increases/decreases: for instance, it is usually the case for an experiment in a towing tank. The velocity of the wing will follow the expression:

$$
U(t) = U_0 + a \Delta t_a g \left( \frac{t - t_a}{\Delta t_a} \right) = U_0 + \Delta U g \left( \frac{t - t_a}{\Delta t_a} \right),
$$

where $U_0$, $a$, $t_a$, $\Delta t_a$ and $\Delta U = U_1 - U_0$ are respectively the wing speed before the acceleration, the acceleration itself, the time when the acceleration starts, the duration of the acceleration and the total velocity variation. The function $g(\zeta)$ is defined as followed:

$$
g(\zeta) = \begin{cases} 
0 & \text{if } \zeta \leq 0 \\
\zeta & \text{if } 0 < \zeta \leq 1 \\
1 & \text{if } \zeta > 1
\end{cases}
$$

The wing motion is composed of three phases. The first phase consists in the flight at a constant velocity $U_0$, then followed by an acceleration or an deceleration phase achieved in a time $\Delta t_a$ and the third one is the flight at a higher/lower velocity $U_1$.

The wake circulation increases following $U(t)$:

$$
\Gamma(y, t) = \Gamma(y) \left( 1 + \Delta U \frac{t - t_a}{U_0} g \left( \frac{t - t_a}{\Delta t_a} \right) \right).
$$

Here we use an acceleration since the instability propagates from the low circulation $\Gamma_0$ region to the higher circulation $\Gamma_1$ region: we can thus observe the end-effect longer due to the inflow/outflow.

The speed of the wing varies from $U_0$ to $U_1$ over a distance $E$. We thus have:

$$
\Delta t_a = \frac{2E}{2U_0 + \Delta U}
$$

The S-D simulation considered here is, at first, a complete rollup of the vortex sheet shed by a wing having an elliptic span loading and flying at velocity $U_0$. The wing then accelerates to the velocity $U_1$ over a distance $E = b/2$. The component of the vorticity field in the flight direction ($\hat{e}_x$, the axial direction) also increases:

$$
\omega_x(y, z, t) = \omega_x(y, z) \left( 1 + \Delta U \frac{t - T_a}{U_0} g \left( \frac{t - T_a}{\Delta T_a} \right) \right).
$$
A component of the vorticity field in the spanwise direction ($\hat{e}_y$) is then also produced corresponding to the temporal variation of the circulation. Starting using the total derivative of $\omega_y(y, z, t)$, we can write

$$\omega_y(y, z, t) = \int \frac{\partial \omega_y}{\partial y} dy + \int \frac{\partial \omega_y}{\partial z} dz + \int \frac{\partial \omega_y}{\partial t} dt. \quad (26)$$

Since the vorticity field is divergence free and since the vertical vorticity component is zero, this gives:

$$\omega_y(y, z, t) = -\int \frac{\partial \omega_x}{\partial x} dx + \int \frac{\partial \omega_y}{\partial z} dz + \int \frac{\partial \omega_y}{\partial t} dt. \quad (27)$$

One further assumes that the spanwise component of the vorticity due to the acceleration is uniform in the vertical direction ($\frac{\partial \omega_y}{\partial z} = 0$) and over the time step $dt$ ($\frac{\partial \omega_y}{\partial t} = 0$). One then obtains:

$$\omega_y(y, z, t) \approx -\int \frac{\partial \omega_x}{\partial x} dx = -\frac{dt}{dx} \int \frac{\partial \omega_x}{\partial t} dt = -\frac{1}{U(t)} \frac{\Delta U}{\Delta t} \int_0^y \omega_x(y', z) dy'. \quad (28)$$

We define the dimensionless time with the characteristic time of the vortex pair during the high velocity regime: $t_0 = \frac{2\pi b^2}{\Gamma_1}$ (region towards which the instability propagates).

A computational region of interest for the T-D simulation corresponds to a region of the S-D simulation moving at $U_1$ as sketched in Fig.13. To make their comparison, we follow a reference cross plane $x_{ref}$: moving in the S-D case and fixed in the T-D case.

The time step of the simulation is fixed to $\Delta t/t_0 = 6.45 \times 10^{-5}$. The mesh size is $h/b = 1/100$ and the regularization parameter is $\sigma = 2h$. The length of the computational box has been chosen long enough to observe the phenomenon: $L_x/b = 15$. The simulation required 20 processors for 13,500 time steps (run for $\sim 125$ hours). We carried out the simulation using $\sim 40M$ grid points.

### 4.2 Numerical setup & initial condition for the time-developing simulation

The initial condition of the T-D simulation is in the same spirit of the S-D simulation. Over the distance $E$ (also fixed to $b/2$), the circulation of the half vortex sheet increases from $\Gamma_0$ to $\Gamma_1$ in space (and not in time as in the space-developing simulation):

$$\omega_x(x, y, z) = \omega_x(y, z) \left(1 + \frac{\Delta U}{U_0} \theta \left(\frac{x}{E}\right)\right). \quad (28)$$

To make the problem periodic, we finally use:

$$\omega_x(x, y, z) = \omega_x(y, z) \left(1 + \frac{\Delta U}{U_0} \theta \left(\frac{x}{E}\right) \theta \left(\frac{L_1 - x}{E}\right)\right). \quad (29)$$

Since the vorticity field is divergence free and since the vertical component of the vorticity is zero, the spanwise vorticity component is obtained from:

$$\omega_y(x, y, z) = -\int \frac{\partial \omega_x}{\partial x} dy. \quad (30)$$
leading to

$$\omega_y(x, y, z) = -\frac{\Delta U}{U_0} \left( g\left(\frac{x}{E}\right) g\left(\frac{L_1 - x}{E}\right) \right)' \int_0^y \omega_x(y', z) dy'. \tag{31}$$

Fig. 14 presents the different time-evolution of the wake circulation for the S-D and for the T-D simulations which corresponds to the wake circulation of a plane moving in the initial condition at the wing speed. The initial condition for the T-D case is thus a good candidate to approximate the S-D case. The distances corresponding to the two regimes have been fixed to $L_0 = 10b$ and $L_1 = 20b$. The other numerical parameters are the same as in the S-D simulation, i.e. the mesh size $h/b = 1/100$, the time step $\Delta t/t_0 = 6.45 \times 10^{-5}$ and the regularization parameter $\sigma = 2h$. The simulation required $\sim 65M$ of grid points and was carried out using 40 processors for 4,800 time steps (it ran 170 hours).

4.3 Analysis of the initial condition for the space-developing simulation: vortex sheet rollup at $Re \Gamma = 10^4$.

Fig. 15 shows a general 3D view of the S-D vortex sheet rollup at $Re = \Gamma_0/\nu = 10^4$. Information about the circulation profile is an important issue in order to model trailing vortices. Many analytical circulation profile formula, also called “vortex models”, and their related velocity profile, have been developed. The most usual models are itemized in details in [6] (see also Table 1 for a summary).

We also connect the 2-D rollup simulations and the 3-D S-D simulations by:

$$x = U_0 t = U_0 t_0 \tau,$$

with $\tau = t/t_0$ the dimensionless time for the 2-D simulation, with $t_0 = b_0/V_0$ (where $V_0 = \Gamma_0/(2\pi b_0)$ is the wake descent velocity. A station at $x/b$ in the 3-D S-D simulation thus has an equivalent dimensionless time equal to

$$\tau = \frac{x/b}{\left(\frac{U_0}{V_0} \frac{2\pi b_0}{\Gamma_0}\right)}.$$
Figure 15: 3-D visualization of a S-D vortex sheet rollup. Isocontour of the vorticity field norm $||\omega||b^2/\Gamma_0 = 8.0$ colored by the axial vorticity component. A set of streamlines are also plotted in the wingtip region, going above/under the wing (orange/cyan lines).
**Table 1:** List of usual vortex models. Also provided, the values of the model parameter(s) fitted on the space-developing rollup simulation at \( x/b = 5.0 \) and \( x/b = 10 \).

<table>
<thead>
<tr>
<th>Vortex Model</th>
<th>( \Gamma(r)/\Gamma_0 )</th>
<th>( a_1/b )</th>
<th>( a_2/b )</th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low Order Algebraic</td>
<td>( \frac{(r/a_1)^2}{(r/a_1)^2+1} )</td>
<td>0.051</td>
<td>/</td>
<td>( r_c = a_1 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.055</td>
<td>/</td>
<td></td>
</tr>
<tr>
<td>High Order Algebraic</td>
<td>( \frac{(r/a_1)^2((r/a_2)^2+2\gamma)}{((r/a_1)^2+\gamma)^2} )</td>
<td>0.058</td>
<td>/</td>
<td>( r_c = a_1 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.064</td>
<td>/</td>
<td>( \gamma = 1.781 )</td>
</tr>
<tr>
<td>Gaussian</td>
<td>( 1 - \exp(-\beta (r/a_1)^2) )</td>
<td>0.068</td>
<td>/</td>
<td>( r_c = a_1 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.073</td>
<td>/</td>
<td>( \beta = 1.256 )</td>
</tr>
<tr>
<td>Jacquin</td>
<td>( \frac{(r/a_2)^2}{((a_1/a_2)^4+(r/a_2)^4)^{1+a/4}(1+(r/a_2)^2)^{1-a/4}} )</td>
<td>0.041</td>
<td>0.27</td>
<td>( \alpha = 0.7 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.048</td>
<td>0.25</td>
<td></td>
</tr>
<tr>
<td>Proctor-Winckelmans</td>
<td>( 1 - \exp\left(-\frac{(r/a_1)^2}{1+\left(\frac{(r/a_1)^2}{a_2/(r/a_2)^4}\right)^{1/p}}\right) )</td>
<td>0.051</td>
<td>0.068</td>
<td>( r_c \simeq (a_1^8/a_2^3)^{1/5} )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.060</td>
<td>0.066</td>
<td>( p = 5 )</td>
</tr>
<tr>
<td>Regularized Kaden-Winckelmans</td>
<td>( 1 - \exp\left(-\left(r/a_1\right)^{3/2}\right)\frac{a(r/a_2)^{1/2}}{1+(a-1)(r/a_2)} )</td>
<td>0.033</td>
<td>0.39(\simeq 0.5\frac{\pi}{4})</td>
<td>( \alpha = 1.78 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.040</td>
<td>0.36</td>
<td></td>
</tr>
</tbody>
</table>
Fig. 16 shows the circulation profiles $\Gamma(r)$ and the tangential velocity profiles $u_\theta(r) = \Gamma(r)/(2\pi r)$ that we have obtained for various cross planes. All profiles have been fitted on the tangential velocity profiles in the least-squared sense without weight function. The maximum tangential velocity slightly decreases by diffusion from $8.2V_0$ to $7.0V_0$. Fig. 17 and Fig. 18 show the tangential velocity profiles and the related circulation profiles for various fitted models at $x/b = 5.0$ and $x/b = 10.0$ where the rollup is almost completed. Table 1 provides the model fitted parameters. We observe that the one-length scale models (i.e. the Low Order Algebraic, the High Order Algebraic and the Gaussian) are not able to correctly capture the circulation profile and thus also the tangential velocity profile. Nevertheless, the best one-length scale model is the Low Order Algebraic one. The two-length scale models (i.e. the Jacquin model and the Proctor-Winckelmanns model) are able to match properly both the maximum tangential velocity and the radius core size $r_c$. It has to be noticed that the “global parameter” of these two models which are assumed to be “universal” (for wake at high Reynolds number and with thin vortex sheet before rollup), was here modified to have a good fit. We here investigate thicker initial vortex sheet and lower Reynolds number: the Jacquin parameter $\alpha$ has been taken to 0.7 and $p = 5$ in the Proctor-Winckelmanns model. Finally, the Regularized Kaden-Winckelmanns model fits better than all one-length scale models but doesn’t quite reach a high enough maximum tangential velocity. For long time, it seems to be close to the Low Order Algebraic model.

Fig. 19 presents the evolution of the vortex core radius $r_c$ for the S-D case and the T-D case. During the formation of the vortex pair, the vortex core quickly increases until $x/b \simeq 5$ ($\tau = 0.16$). As foreseen, the S-D simulation and the T-D simulation are very similar in terms of $r_c$ and $u_{\theta,\text{max}}$.

Fig. 20 and Fig. 21 show the axial velocity. We analyse the value of the maximum axial velocity (positive and negative) in Fig. 20 and provide cross-views in Fig. 21. It is interesting to note that both are of the same order of magnitude. We find a peak of axial velocity deficit (i.e. towards the wing) in the vortex core roughly valued to $0.5V_0$ (when the rollup is completed and increasing shortly after). Moreover, an axial velocity in the opposite direction (away from the wing) surrounds the vortex core at the peak value of roughly $0.4V_0$. 

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Figure 16: Circulation profiles (a) and tangential velocity profiles (b) at various distances in the wake.
Figure 17: Tangential velocity profiles for various vortex models (a) at $x/b = 5.0$ ($\tau = 0.16$) and (b) at $x/b = 10.0$ ($\tau = 0.33$).
Figure 18: Circulation profiles for various vortex models (a) at $x/b = 5.0$ ($\tau = 0.16$) and (b) at $x/b = 10.0$ ($\tau = 0.33$).
Figure 19: Evolution of the vortex effective core size $r_c$ (a) and of the maximum tangential velocity $u_{\theta,\text{max}}$ (b) for the T-D simulation (tick line) and for the S-D simulation (thin line).

Figure 20: Evolution of the maximum axial velocity (solid) and the minimum axial velocity (dash) with respect to the distance in the wake.
Figure 21: (a) Slice of the axial component of the vorticity field $\omega_x t_0$. (b) Slice of the axial component of the velocity field $u_x/V_0$. 
\( x/b = 4.0 \ (\tau = 0.13) \)
Figure 21: [Cont’d]
\( x/b = 12.0 \ (\tau = 0.39) \)

\( \text{Figure 21: } \) [Cont’d]
4.4 Comparison and analysis of the space-developing and of the time-developing simulations

**Global dynamics**  Fig. 22 presents the 3-D view of the S-D evolution of the accelerated wing and the resulting end-effect. While the wing accelerates, the vorticity generated by the acceleration rolls up around the two wake vortices. This creates an external deformation of the vortices. Afterwards, a helical instability develops that moves towards the stronger vortex and irremediably deforms the wake vortices. Fig. 23 shows the 3-D view of the end-effect for the T-D simulation. The same physics occur.

**Visualization of the end-effect**  Fig. 24 shows a zoom, in the propagation region, of the vorticity isocontour at a low level of vorticity norm. In the S-D case, the end-effect appears to be more confined. The reason could be the fact that, in this case, an axial velocity (in the $\hat{e}_x$ direction) surrounds the vortex and then convects, in the wake, the external deformation which propagates in the opposite direction. However, the pressure wave is not affected by the slowing down of the external helical instability. Fig. 25 shows two isocontours of vorticity norm for higher levels in order to visualize the inner vortices. A pressure wave propagates along each vortex which creates a strong axial velocity, destabilizes each vortex and allows the development of a helical instability.

**Propagation of the instability**  Fig. 26 compares the evolution of the axial velocity front ($u_a$). This velocity is slightly higher in the S-D case : $u_a/\theta_{\max} = 1.30$ instead of 1.20 for the T-D case. The difference might partially come from the additional axial velocity in the vortex core, valued at $\simeq 0.5V_0$. However, the global structure of the axial velocity is quite similar between the two cases, as shown in Fig. 27. The simulations were carried out at relatively high Reynolds number; the vortex core diffusion doesn’t play much of a role and the front location evolves quasi linearly in time.

As done in the first section, we can analyze the axial component of the vorticity perturbation field $\omega'_x$. Fig. 28 compares the isocontours of this field between the T-D simulation and the S-D simulation. The vorticity base fields $\omega_x^{(0)}$ are here taken as the vorticity field at the center slice of the high speed region for the T-D case, and as the vorticity field at the same time-evolving slice but for a rollup simulation at high speed regime for the S-D case. It confirms the conclusion of Mo et al. [12] : an axisymmetric instability is at first created which magnify the vortex core. This instability is followed by a helical one (the positive and the negative levels of vorticity perturbations are interlaced) that leads to the loss of the vortex core coherence.
Figure 22: 3-D view of the wake for the space-developing simulation of an accelerated wing. Isocontour of the vorticity norm $\|\omega\| b^2/\Gamma_1 = 8$ colored by the axial vorticity component.
Figure 22: [Cont’d]
\[ t u_{\theta, \text{max}} / b = 2.9 \left( t \frac{\Gamma_1}{2\pi b_0} = 0.45 \right) \]

\[ t u_{\theta, \text{max}} / b = 3.3 \left( t \frac{\Gamma_1}{2\pi b_0} = 0.52 \right) \]

Figure 22: [Cont’d]
Figure 23: 3-D view of the wake for the time-developing simulation of an accelerated wing. Zoom of the vorticity norm isocontour $||\omega|| b^2 / \Gamma_1 = 8$ colored by the axial vorticity component.
\[ tu_{\theta, \text{max}} / b = 1.0 \left( t \frac{\Gamma_1}{2 \pi b_0} = 0.15 \right) \]

\[ tu_{\theta, \text{max}} / b = 1.7 \left( t \frac{\Gamma_1}{2 \pi b_0} = 0.26 \right) \]

Figure 23: [Cont’d]
\[ tu_{g, \text{max}} / b = 2.7 \left( t \frac{\Gamma_1}{2 \pi b_0} = 0.41 \right) \]

\[ tu_{g, \text{max}} / b = 4.0 \left( t \frac{\Gamma_1}{2 \pi b_0} = 0.62 \right) \]

**Figure 23**: (Cont’d)
\[ \frac{t u_{\theta, \text{max}}}{b} = 0.33, \]
\[ \frac{t u_{\theta, \text{max}}}{b} = 1.2, \]
\[ \frac{t u_{\theta, \text{max}}}{b} = 2.0, \]
\[ \frac{t u_{\theta, \text{max}}}{b} = 2.8. \]

**Figure 24:** Top view of the vorticity field isocontour \( ||\mathbf{\omega}||^2 / \Gamma_1 = 8 \) colored by the axial vorticity component. Comparison of the instability evolution between the space-developing (S-D) and the time-developing (T-D) simulations for various times. The length of the visualization box is \( 4b \) centered relatively to \( x_{\text{ref}} \).
Figure 25: Top view of the vorticity isocontours colored by the axial vorticity component $||\mathbf{\omega}|| b^2 / \Gamma_1 = 40$ (low opacity) and $||\mathbf{\omega}|| b^2 / \Gamma_1 = 60$ (high opacity). Comparison between the space-developing simulation (left) and the time-developing simulation (right) at the time $tu_{\theta_{\text{max}}} / b = 2.8$. Also shown are the slices used Fig. 28.
Figure 26: Evolution of the non-zero axial velocity front for the space-developing simulation (dash) and the time-developing simulation (solid). The thin lines corresponds to the least-squared fitted lines.

Figure 27: Cross plane view of the axial velocity field at $y/b = 0.5\pi/4$ for (a) the space-developing simulation and (b) the time-developing simulation at the time $tu_{\theta,max}/b = 2.8$. 
Figure 28: Ten equally spaced levels of the axial vorticity isocontour $\omega_x b^2 / \Gamma_1$ from $-5.5$ to $5.5$, at $tu_{0,\text{max}}/b = 2.8$, for various cross planes: (a) $x/b = x_{\text{ref}}/b - 3$, (b) $x/b = x_{\text{ref}}/b - 2.5$ and (c) $x/b = x_{\text{ref}}/b - 1$. Positive levels are plotted in red and the negative levels are plotted in blue. The circle with $r = r_c$ is also plotted in black.
5 Conclusion

The VIC-PFM method was shown to be suitable to simulate the dynamics of end-effects phenomena, both for the time-developing (T-D) and the space-developing (S-D) simulations. We observed comparable results to those of CNRS/IRPHE. The propagation speeds are found to be in good agreement with the theoretically predicted values. The same dynamics were observed for each simulated case, confirming the observations of Moet et al.[12]: at first an axisymmetric perturbation, corresponding to a pressure wave, propagates along the vortex core; if the resulting axial velocity is strong enough, as in the configurations investigated here, this wave can destabilize the vortex and a helical instability then develops.

The S-D simulation with wing acceleration points out a difference compared to the time-developing simulation in terms of propagation speed. The difference however remains minor and doesn’t modify the dynamics.

The net result of the S-D wake rollup has been analyzed. It appears that the Proctor-Winckelmans and the Jacquin models are both good candidates to model wake vortices. Due to the 3-D rollup, an axial velocity (towards the wing) is also created, valued at $0.5V_0$.

Note however that the present S-D simulation, and the resulting 3-D rollup, underestimates the intensity of the axial velocity within the vortex core. It would likely be much higher considering a 3-D rollup with momentum deficit just behind the wing corresponding to the boundary layers and profile drag. This was not done in the present work; yest it constitutes an interesting follow up. It also can explain the different behaviour of the “end-effect” associated with an acceleration or a deceleration of the wing, as observed experimentally.
References


