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Roll-up of a temporally-evolving wing wake in presence of jets

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Roll-up of a temporally-evolving wing wake in presence of jets
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Abstract

The temporal evolution of a propelled wing wake, including two jets, is investigated at high Reynolds number by means of Large Eddy Simulation (LES). The present work is a continuation of [4] in which the temporal evolution of a wing wake with velocity deficit was recently investigated. The same wake model is used. The longitudinal vorticity field is given by Prandtl lifting line theory applied to an elliptic wing. The effect of the boundary layers is taken into account by adding a velocity deficit inside the regularized vortex sheet. Two jets are then added to simulate the wake of a propelled wing. The jet configuration is determined so as to represent as realistically as possible a cruise configuration. The main characteristics are a large initial jet/wing-tip distance and a relatively large jet to vortex strength ratio, the jet strength being determined so as to compensate the total drag of the wing.

The efficient combination of Vortex-In-cell and Parallel Fast Multipole methods, the VIC-PFM, is used. The jets impose a high numerical resolution compared to the case with wing wake alone. The present configuration is compared to the latter. An additional configuration without jets, but with the same high numerical resolution as for the propelled case, is also considered in order to determine the effect of the numerical resolution.

The global dynamics are shown to be similar to the ones obtained without jets. This is due to the large initial jet/wing-tip distance. The same successive instabilities develop, generating vortical structures and inducing a deformation of the vortex core. The jet fluid is entrained by the vortex induced velocity during the roll-up. It is wrapped around it but does not penetrate the core. When surrounding the vortex core, the jet velocity excess has significantly decayed. The vortex structure is thus not significantly affected.

The comparison of the two configurations without jets shows that, although both numerical resolutions are high, the highest one provides addition details concerning the vortex structure.
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Introduction

Aircraft wake vortices constitute a crucial issue for air traffic safety and optimal management. The constant air traffic increase and the actual congestion of all major airports recently led to large research efforts. The extensive investigations dedicated to wake vortices aim, at first, gaining a thorough understanding of the complex physical phenomena and, second, at looking for some ways to control and/or alleviate their formation and downstream evolution. The continuous development of experimental, numerical and theoretical methods, together with the increase in computer capacities, enable to focus on more and more realistic configurations. A recent example concerns the effect of the engine jets on the wake roll-up and downstream development. The interest concerns both points mentioned above. First, because in air traffic management, the wakes of interest are generated in presence of the aircraft engine jets. Second, because engine jets are candidates to be used to weaken or destroy more rapidly the wake vortices. Section 1 presents a review of previous work and present knowledge concerning the interactions of wake vortices with jets, as well as the remaining issues.

The present work is a contribution to the FAR-Wake project, dedicated to open wake vortex issues, including the effect of engine jets. The time-evolution of a propelled-wing wake (including two engine jets) is thus investigated by means of temporal Large Eddy Simulation (LES). This work is a continuation of [4] in which the roll-up of a wing wake with velocity deficit was investigated. The same approach and methods are used. The combination of Vortex-In-Cell and Parallel Fast Multipole methods, the VIC-PFM method, which proved high efficiency to simulate wake vortex flows, is used. The temporal LES is carried out at large Reynolds number ($Re_T = \Gamma_0/\nu = 10^6$, $\Gamma_0$ being the initial half plane circulation and $\nu$ the fluid viscosity). The simulation is initialized using the same wing wake model with velocity deficit as that used in [4], with two additional jets. The parameters are determined so as to represent as realistically as possible a cruise configuration. The initial jet/vortex separation distance is thus relatively high and the jet thrust compensates for the wing total drag. The numerical method and the initial wake model are described in Section 2.

The time-evolving dynamics of the propelled-wing wake are investigated and compared to the case without jets. The latter was analyzed in details in [4]. However, in the present case, the numerical resolution (required to reasonably capture the jets) is twice higher than in the latter. In order to determine the effect of the numerical resolution on the results without jets, a configuration without jets has also been simulated with the same high numerical resolution as for the propelled-wing wake. The qualitative and quantitative analysis of the jet effects, including the comparison to the two configurations without jets, is presented in Section 3.
1 Context

The present work is a continuation of [4], where the roll-up of a temporally-evolving wing wake with velocity deficit due to boundary layers was investigated by means of LES. A detailed analysis of the near-field to far-field (after roll-up) development of the wing wake was performed. A review of previous work and present knowledge concerning near-field wing-tip vortices was also presented. The information is not repeated here and the interested reader is referred to [4].

In the present work, a propelled wing wake is considered, corresponding to the same wake with velocity deficit as in [4], with two added jets to account for the presence of the engine jets. To the knowledge of the authors, this is the first numerical investigation of such a “complete” wake, the computational requirements being large. Previous numerical investigations focussed on single jet/vortex interactions, and complete wing wakes in presence of jets have been investigated experimentally.

The dynamics of aircraft wake vortices in presence of jets are determined by a large number of parameters: the relative position of the jets with respect to the wing tips, the jet to vortex strength ratio, the jet to freestream angle, the Reynolds number and the jet to freestream temperature ratio for the main ones. Thus, although a certain number of investigations have been carried out in the past, the cross-comparison is made difficult. The understanding of the various mechanisms and of the effect of each parameter is thus not complete. The objective of the FAR-Wake project is to determine the present knowledge and to carry out related numerical and experimental parametric investigations to gain a good general understanding as well as insights in the effects of the different parameters.

So far, a synthesis on single vortex/cold jet interactions, including a short review of previous work and present knowledge as of the start of the project (2005) as well as the synthesis of the work performed within FAR-Wake has been delivered [11]. One clear result concerns the effect of the jet to freestream temperature ratio. The analysis of different experimental test campaigns showed that the temperature of the jet did not affect the dynamics [9]. Cold and hot jet studies can thus be used and compared in the same way.

Two main parameters appear to control the interaction between a jet and a single vortex: the initial separation distance and the relative strengths of the jet and of the vortex (see [11] and references therein). The latter is evaluated using the parameter $R$: 

$$ R = \frac{\rho \int_A U (U - U_\infty) dA}{\rho \Gamma^2} $$

where $\rho$ is the fluid density, $U$ is the axial velocity, $U_\infty$ is the freestream velocity, $\Gamma$ is the vortex circulation and $A$ is the cross-section. The lower the separation distance and the higher the parameter $R$, the larger the effect of the jet on the vortex dynamics. Basically, large separation distances and relatively large $R$ parameters correspond to cruise configurations, where the wing tip vortex interacts with the engine jet. Low separation distances correspond to take-off and landing configurations, where the flap vortex interacts with the engine jet. In the former case, the thrust of the jet and thus the parameter $R$ is large, while in the later case it is relatively low.

In well-separated cases, the dynamics are dominated by the entrainment of the jet fluid by the vortex (see [11] and references therein). The jet is intensively stretched and deformed. Three phases can be observed. In the first one, the jet fluid is entrained by
the vortex rotational velocity: it is pulled closer to the vortex core and gets wrapped around it. In the second phase, the deformed jet is wrapping around the vortex core, generating secondary vortical structures. The rapidity of formation and the intensity of these azimuthal vortical structures depends on the initial separating distance and on the $R$ parameter. In the third phase, these vortical structures have significantly decayed. The remaining flow consists of a vortex core surrounded by weak small-scale vortical structures. The main effect of the jet on the vortex is the reduction of its maximum vorticity. For large enough R parameters, the vortex core is also enlarged. For these large initial separation distances, the jet fluid does not penetrate the vortex core and only the axial velocity in the periphery of the vortex is increased.

For low initial separation distances however (from 2.5 to 4.5 vortex core diameters, the latter being the relevant length scale in this case [11]), a small part of the jet fluid penetrates into the vortex core. In these cases, the axial velocity deficit in the wing tip vortex core is also significantly reduced (and can turn to a velocity excess in some cases). The effects of the jet, described above for large initial separation distances, are enhanced and occur earlier.

An additional conclusion of the experimental investigations in [9] concerns the downstream position of the vortex center. When reducing the initial jet/vortex separation distance, it is shown that the lateral and vertical displacements of the vortex, inward and downward from the wing tip respectively, tend to be enhanced.

Although the different investigations considered in [11] are in good general agreement, one notes that the configurations are significantly different from one to another. As pointed out in [11], the dynamics of a single simplified (Gaussian) rolled-up vortex interacting with a turbulent jet (as investigated numerically in [12]) is different from the interaction between the ones of a complete wing wake vortex sheet rolling-up in presence of jets (as investigated experimentally in [9]). Indeed, the roll-up phase itself is strongly affected by the jets and is not taken into account in the former case. This point was also stressed in [10], in which a numerical simulation similar to [12] was carried out with the objective to compare to the experimental results of [9]. The conclusion was that these two configurations were not directly comparable due to the absence of a counter-rotating vortex in the numerical case, hence, the absence of induced motion on the single vortex/jet system.

Besides these investigations carried out in the framework of the FAR-Wake project, a comparative experimental/numerical study was performed in [8]. The numerical approach is different from [12] and [10]: in a first phase, the flow past a half-wing model was computed and, subsequently, the output of this preliminary simulation was used as the inflow for the simulation of the downstream development of the wake. The experimental and realistic conditions (including the roll-up of the complete half-wing wake) were thus well represented and the agreement with the corresponding experiments was reasonable. However, due to the computational cost of such a simulation, the maximal downstream position investigated was low ($x/c = 4$ with $c$ being the chord length of the rectangular half-wing).

Considering the advantage of a careful numerical simulation in terms of physical analysis, the numerical simulation of the complete roll-up and the further downstream evolution of a complete propelled-wing wake is thus of great interest. In the present work, a cruise configuration is considered; hence, a relatively large initial separation distance and a high parameter $R$. Due to the large computational cost of such a numerical simulation, one
configuration is investigated in details and a parametric study is not performed. The objective is to focus on the physical mechanisms in order to improve our understanding of a realistic configuration and to give more details on some phenomena observed experimentally. In the next section, the numerical method and initial condition used in the present work are described in details.

2 Method

2.1 Three-dimensional combination of Vortex-in-Cell and Parallel Fast Multipole methods

In the present work, three-dimensional temporal (longitudinally periodic) Large Eddy Simulations (LES) are performed. A hybrid approach, combination of Lagrangian and finite difference methods, is used: the Vortex-In-Cell Parallel Fast Multipole method (VIC-PFM code). In this new combination of Vortex-In-Cell and Parallel Fast Multipole methods, presented in details in [2], the vorticity form of the incompressible Navier-Stokes equations is solved. The numerical method has already been described in more details in [4] and the information is thus not repeated here. Particular to the present implementation of this procedure is the treatment of the boundary conditions on the grid, using the Parallel Fast Multipole method (PFM). With this approach, the unbounded domain condition can be ensured accurately on a relatively small grid: only as large as the vorticity field itself. This allows for a significant reduction of the computational cost. Furthermore, the method can be parallelized using a domain decomposition approach: the PFM code, which has a global view of the whole field, is then used to obtain proper boundary conditions on each subdomain [13, 2, 14], leading to a very efficient numerical method. The LES modeling is here done using a multiscale subgrid-scale model: the model solely acts on the small scale part of the LES field. We here use our version of the Regularized Variational Multiscale (RVM) model [1]. The advantage of such model is that it preserves the inertial range while providing dissipation at the high wavenumbers: it is only active during the complex phases of the flow, while remaining inactive in the laminar and/or well-resolved regions.

The temporal evolution of the wake is investigated in the present work. Temporal LES are therefore carried out using periodic boundary conditions in the longitudinal direction. The length of the computational domain is $L_x/b = 0.5$, where $b$ is the wingspan of the wing. In the transverse directions, open domain conditions are used as mentioned above. The initial conditions, described in details in the next section, are two-dimensional vorticity fields extruded in the periodic direction. No perturbation was added. Due to the method used, the number of Lagrangian vortex particles and the corresponding size of the Eulerian grid growths as the simulation proceeds (as the wake is rolling-up). In order to optimize the efficiency of the numerical simulation, the computational parameters are adapted during the simulation. In the following, the range (initial/minimum and final/maximum values) are given for each parameters. The simulations were performed on 10 to 40 Opteron processors at 2.6 Ghz. The time step was adapted during the simulation in order to capture the large flow variations occurring in the early stages of the thin vortex sheet roll-up and jet development, and to optimize the computational time subsequently. Thus, the time steps ranged from $\Delta t/t_0 = 4 \times 10^{-5}$ to $1 \times 10^{-4}$ (where $t_0 = b_0/V_0$ is the characteristic time of the vortex pair defined in Section 3). The number of particles and of Vortex-In-Cell grid points were initially about $4 \times 10^6$ and $8 \times 10^6$, respectively, and about $50 \times 10^6$ and $96 \times 10^6$, by
the end of the simulation (corresponding to a dimensionless time and downstream position - defined in Section 3 - of $\tau = 0.75$ and $x/b = 22.8$).

### 2.2 Initial conditions: description of the wake model

The objective of the present work is to investigate the effect of two engine jets on the roll-up of the wake generated by an aircraft. The approach consists in modelling the wake generated by a wing in presence of two jets, and simulating its time-evolution. This work is a continuation of [4], in which the same approach was used to investigate the effect of the velocity deficit due to the boundary layers on the wake roll-up. A model was developed for the wing wake (the initial condition) to take into account the velocity deficit. In the present work, the same wake model is used.

The streamwise vorticity component is given by the lifting line theory applied to an elliptically loaded wing. The theory consists in replacing the lifting wing of lift distribution $l(y)$ by a vortex line defined by its span loading, $\Gamma(y)$, using the relation: $l(y) = \rho U_\infty \Gamma(y)$ ($\rho$ being the fluid density and $U_\infty$ the flight speed). For an elliptically loaded wing, one obtains:

$$\Gamma(y) = \Gamma_0 \sqrt{1 - \left(\frac{y}{b/2}\right)^2} \quad (2)$$

where $\Gamma_0$ is the half plane total circulation. It characterizes the wake together with the vortex spacing (after roll-up), $b_0$:

$$\int_{-b/2}^{b/2} \Gamma(y) dy = \Gamma_0 b_0 \quad (3)$$

with $b_0 = \frac{\pi}{4} b$ for elliptical loading. Considering the definition of the total wing lift, and the lift coefficient, respectively:

$$L = \int_{-b/2}^{b/2} l(y) dy = \rho U_\infty \int_{-b/2}^{b/2} \Gamma(y) dy = \rho U_\infty \Gamma_0 b_0 \quad (4)$$

$$C_L = \frac{L}{\frac{1}{2} \rho U_\infty^2 S} = \frac{2 \Gamma_0 b_0}{U_\infty S} \quad (5)$$

with $S = \frac{b^2}{Ar}$ the wing surface ($Ar$ being the wing aspect ratio), the characteristics of the wake can be expressed as a function of the wing characteristics:

$$\Gamma_0 b_0 = \frac{1}{2} U_\infty b^2 \frac{C_L}{Ar} \quad (6)$$

One also obtains the mutually induced descent velocity of the vortex pair:

$$V_0 = \frac{\Gamma_0}{2 \pi b_0} = \frac{U_\infty}{4 \pi} \left(\frac{b}{b_0}\right)^2 \frac{C_L}{Ar} \quad (7)$$

and thus, for an elliptical loading,

$$\frac{V_0}{U_\infty} = \frac{4}{\pi^3} \frac{C_L}{Ar} \quad (8)$$
In the present work, the wing characteristics are:

\[ C_L = 1.5 \quad (9) \]
\[ Ar = 7.5 \quad (10) \]

thus, \( \frac{C_L}{Ar} = 0.20 \) and \( V_0/U_\infty = 0.026 \) The Reynolds number is taken as:

\[ Re_\Gamma = \frac{\Gamma_0}{\nu} = 10^6. \quad (11) \]

The corresponding shed vorticity field is a vortex sheet with circulation per unit length:

\[ \gamma(y) = -\frac{d\Gamma}{dy}(y) \quad (12) \]

This singular vortex sheet is then regularized using a Gaussian kernel in order to produce a regular vorticity field:

\[ \omega_\sigma(y, z) = \frac{1}{\pi\sigma^2} \int_{-b/2}^{b/2} \exp \left( -\frac{(y - y')^2 + z^2}{\sigma^2} \right) \gamma(y') dy' \quad (13) \]

where \( \sigma \) is the regularization parameter, set in the present work to \( \sigma/b = 1/75 \), still giving a fairly thin vortex sheet. The grid resolution, \( h/b = 1/400 \), significantly higher than in [4] (where we used \( h/b = 1/200 \)), is determined to obtain a reasonable resolution of the thin shear layer of the two jets (see below). The resulting longitudinal vorticity field is presented in figure 1a.

The velocity deficit (corresponding to spanwise and vertical vorticity components) is determined in order to give the same induced drag to profile drag ratio as for an aircraft in cruise (the induced drag contributing to 40 to 50% of the total drag [5]). The velocity deficit is distributed elliptically in the spanwise direction:

\[ U_w(y, 0) = U_{w0} \sqrt{1 - \left( \frac{y}{b/2} \right)^2} \]

\( U_w = U - U_\infty \) is a velocity deficit, with \( U \) the axial velocity component and \( U_\infty \) the freestream velocity. The parameter \( U_{w0} \) determines the magnitude of the velocity deficit. The axial direction, \( x \), is defined positively downstream of the wing: the velocity deficit, \( U_w \), is thus negative. It is then regularized to obtain a 2-D axial velocity field. The velocity deficit is thus defined by two parameters: its magnitude, \( U_{w0} \), and its thickness (set by the regularization parameter \( \sigma \)).

Following [6] and references therein, the induced drag is obtained by computing the cross-flow kinetic energy per unit length of the initial vorticity field described above:

\[ D_i \sim E_{c0} = \frac{1}{2} \rho \int_A (v^2 + w^2) dA = \frac{1}{2} \rho \int_A \psi_x \omega_x dA \quad (14) \]

where \( v \) and \( w \) are the velocity components in the \( y \)- and \( z \)-directions, \( A \) is the cross plane, \( \omega_x \) is the \( x \)-component of the vorticity and \( \psi_x \) is the stream-function, satisfying the Poisson equation \( \nabla^2 \psi_x = \frac{\partial^2 \psi_x}{\partial y^2} + \frac{\partial^2 \psi_x}{\partial z^2} = -\omega_x \).

The total drag is the sum of the induced drag and of the “profile” (viscous) drag. In the configuration investigated here, the velocity deficit is a direct consequence of the
development of the boundary layers on the airfoil. The velocity deficit must be defined such that the corresponding profile drag contributes to 50 to 60% of the total drag. For the spatially-evolving wake, the “profile” drag is obtained as:

\[ D_p = \rho \int_A (U_\infty - U) U \, dA \]  

(15)

with \( U_\infty \) the freestream velocity and \( U \) axial velocity. In the present case, the relative velocity deficit is as \( U_w = -(U_\infty - U) \), and thus, the profile drag is obtained as:

\[ D_p = -\rho \int_A (U_\infty + U_w) U_w \, dA \]  

(16)

In order to determine \( U_{w0} \), the fact that the initial flow field corresponds to the one shed at the trailing edge of the generating wing is considered. Concerning the axial velocity field, the no-slip condition imposes the velocity to decrease to zero at the trailing edge (center line of the initial vortex sheet). Thus, from this point of view, the natural value for \( U_{w0} \) is \(-U_\infty\), corresponding to \( U = 0 \).

Concerning the regularization parameter, the same as for the vorticity field is used to obtain the axial velocity field. Thus, the boundary layers have roughly the same thickness as the vortex sheet shed by the wing (from lifting line theory). This set of parameters leads to an induced drag, \( D_i \), and a profile drag, \( D_p \), contributing respectively to 42.4% and 57.6% of the total drag, which is in fair agreement with the respective contributions in cruise. For the results and detailed analysis of such a configuration at \( Re_\Gamma = 10^4 \) and \( 10^6 \), the reader is referred to [4]. In the present work, the Wing Wake case of [4] is compared to the Propelled Wing Wake case, with jets.

Two jets are thus added to the wake model. The geometry of the jets aims at reproducing as much as possible a realistic configuration of an aircraft in cruise, also taking into account the computational constraints. Indeed, since the present numerical method uses a uniform mesh of size \( h \), and since a realistic jet shear layer thickness is very small compared to the wingspan, the resolution required is high (much higher than that required for a wing wake alone, as done in [4]). Thus, a trade-off is done between the computational resolution of the jet and the realism of the configuration (initial jet geometry), leading to \( h/b = 400 \).

Initially, the two jets have a “top hat” velocity profile obtained using a hyperbolic tangent function as done in previous numerical investigations of single vortex/jet interactions [12, 7, 10]. The axial velocity is given by \( U = U_\infty + U_j \) with:

\[ U_j(r) = \frac{1}{2} U_{j0} \left( 1 - \tanh \left[ \frac{1}{4} \frac{R_j}{\theta} \left( r - \frac{R_j}{r} \right) \right] \right) \]  

(17)

where \( R_j \) and \( U_{j0} \) are the jet radius and jet maximal velocity, and \( \theta \) is the jet momentum thickness defined as follows:

\[ \theta = \int_0^{\infty} \frac{U_j(r)}{U_{j0}} \left( 1 - \frac{U_j(r)}{U_{j0}} \right) \, dr \]  

(18)

The jet radius and momentum thickness are set to \( R_j/b = 0.0375 \) and \( \theta/R_j = 0.1 \), respectively.


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Table 1: Summary of the conditions of the two configurations investigated.

Considering the geometry of the two jets, the maximum velocity, $U_{j0}$, is determined so that the corresponding thrust compensates the total drag, sum of the induced drag and profile drag ($D_{tot} = D_i + D_p$), as it is the case in cruise configurations. Since the two jets have the same characteristics, the total thrust of the configuration is given by:

$$T_{tot} = 2 T_j = 2 \rho \int_A U (U - U_\infty) \, dA = 2 \rho \int_A (U_j + U_\infty) U_j \, dA.$$  \hspace{1cm} (19)

Equating $D_{tot}$ with $T_{tot}$ leads to a maximum jet velocity of $U_{j0}/U_\infty = 0.85$, thus corresponding to 1.85 when adding the freestream velocity, $U_\infty$. The resulting $R$ parameter is:

$$R = \frac{\rho \int_A (U_j + U_\infty) U_j \, dA}{\rho \Gamma^2} = 0.82$$  \hspace{1cm} (20)

The corresponding axial velocity and transverse vorticity fields are presented in figure 1(b,c), showing the position of the two jets. The center of each jet is positioned at a distance of 0.2 $b$ laterally and 0.058 $b$ vertically from the center of the wake vortex sheet, which is similar to a realistic two-engines aircraft configuration.

In the next section, the obtained dynamics are analyzed and compared to those obtained for the configuration without jets. Since the configuration with jets requires a higher numerical resolution than the wing wake alone, an additional simulation of the later was performed, with the same high resolution as for the former, in order to determine the effect of the numerical resolution on the results without jets. Table 1 sums up the parameters of these two configurations.

### 3 Results

The dynamics are analyzed in this section. The effect of the jets on the wake is determined by comparing to the configuration without jets (already analyzed in details in [4]). For the time-evolution, the non-dimensional time is used:

$$\tau = t / t_0 \quad \text{with} \quad t_0 = \frac{b_0}{V_0} = \frac{2 \pi b_0^2}{\Gamma_0}$$

The corresponding downstream position of the wake, $x/b$, is obtained from $x = t U_\infty$, giving

$$\frac{x}{b} = \left( \frac{\tau}{2} \right)^2 \frac{\Delta v}{C_L} \tau = 30.4 \tau$$

In the present work the maximum dimensionless time considered is $\tau = 0.75$, corresponding to a downstream position of $x/b = 22.8$. 

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3.1 Qualitative dynamics

The same features are observed as in the configuration without jets [4]. The global dynamics are very similar. The three successive instabilities described in details in [4] occur. In the present case, a high Reynolds number is considered, \( Re_\Gamma = 10^6 \) (corresponding to the largest Reynolds number investigated in [4]). The instabilities thus develop rapidly.

Figure 2 shows the three-dimensional flow field obtained at \( \tau = 0.04 \) for the two configurations (same high numerical resolution, \( h/b = 1/400 \)). The velocity deficit instability (see [4]), in the central part of the wake, is well developed and leads to vortical structures and turbulent motions. The longitudinal instability of the thin vortex sheet is also already developed, both in the periphery of the tip vortices and in the part situated below the vortex cores (and being wrapped and stretched). Secondary vortical structures are being generated and wrapped around the tip vortices. Note that the wavelength of the longitudinal instability that led to the generation of these secondary structures is identical in both configurations. This figure shows the similarity between the two configurations.

In the propelled wing wake case, however, the turbulent development of the jets tends to enhance the break-up of the vortex sheet into small-scale structures. Also, initially, the presence of the jets locally increases the level of the transverse vorticity component (figure 1c). Recalling that the stretching of the later is at the origin of the vortex sheet instabilities development during its roll-up [4], the enhancement of these instabilities in the propelled wing case is expected. Also note that this is observed in the development of the secondary vortical structures, which is slightly more advanced in the case with jets.

Since the presence of the jets leads to different vorticity levels, it is useful to examine vorticity fields rather than at discrete isosurfaces. Indeed, the same dynamics could occur but with different vorticity levels. Figures 3, 4, 5 present the axial vorticity field, transverse vorticity field and axial velocity field, respectively, at \( \tau = 0.08 \) for the three configurations: propelled wing wake, wing wake (same high numerical resolution, \( h/b = 1/400 \)) and wing wake (\( h/b = 1/200 \)). Different interesting results are obtained.

First, let’s compare the two wing wake configurations with different numerical resolutions. Figures 3[b,c] and 4[b,c] show that the two simulations are globally similar. As expected, with the finest mesh, the instabilities result in smaller scales which are not captured in the second case (\( h/b = 1/200 \)). Concerning the vortex core, the axial vorticity fields are also similar (figure 3[b,c]). However, some additional details are captured with the finest mesh. Figure 4b shows that transverse vorticity components are present inside the cores. Also, the velocity deficit has penetrated inside the vortex core but is not concentrated in the vortex center at \( \tau = 0.08 \) (figure 5b).

Comparing the cases with and without jets shows strong similarities (figures 3[a,b], 4[a,b] and 5[a,b]). At this time (\( \tau = 0.08 \)), the jet axial velocity is being entrained by the vortex but is not yet wrapped around it. The vortex core region is thus not yet affected by the jet. However, the effect of the jets on the vortex sheet is observable. The jets enhances the break-up into small-scale structures induced by the vortex sheet instabilities. The structures generated are more important and of smaller scales in the jet region (figures 3a and 4a) compared to the case without jets (figures 3b and 4b). These features are also illustrated by the three-dimensional visualization presented in figure 6.

The later evolution is presented in figure 7, in which the longitudinally averaged velocity fields are shown. The jet velocity is being wrapped around the tip vortices. When
the jet velocity is surrounding the vortex core (figure 7c), it has significantly decayed and does not penetrate into the core. This is in agreement with previous investigations of configurations with large initial jet/vortex distances (see section 1). Figure 8, shows the corresponding axial vorticity fields. The roll-up thus results in two vortices with velocity deficit concentrated into the cores, the jets velocity excess being significantly decayed in their external periphery (see next section for more quantitative details). Figure 9 shows the three-dimensional flow-field at \( \tau = 0.75 \) (the latest time of the simulation). The jet fluid is no longer discernable from the surrounding turbulence. The surrounding small-scale structures are still relatively important and of significant intensity. Vortical structures are also remaining. Similarly to the configuration without jets ([4]), the roll-up with velocity deficit and jets results in two tip vortices deformed helically due to the interactions with the secondary structures and the presence of axial velocity in the cores. The time-evolution of the vortex structure is analyzed in more details in the next section.

3.2 Quantitative dynamics and vortex structure

First, the time-evolution of some global wake characteristics are discussed. Figure 10a compares the trajectories of the two tip-vortices for the configurations with and without jets. The trajectories are globally very similar, the turbulent motion induced by the instabilities and the presence of the jets causes some scattering but no general effect of the jets on the trajectory is observed. In the present configuration it is not surprising considering that the initial jet/vortex separation distance is large and that the jet effect on the downstream position of the vortex cores is significant when the separation distance is short (see Section 1).

The effective vortex core size is presented in figure 10b for the two configurations. At first sight, it is smaller after roll-up with the jets than without. However, when comparing the wing wake simulated with the same high numerical resolution as the configuration with jets (not shown because only available for low dimensionless times), the vortex core size evolution are very similar. Thus, the vortex core size difference pointed out in figure 10b (of 0.009 \( b \)) is partially attributed to the numerical resolution (0.005 \( b \) and 0.0025 \( b \) for the wing wake alone and propelled-wing wake configurations, respectively). Indeed, since the jet fluid does not penetrate the vortex core and since the jet velocity has significantly decayed when it is wrapped around it, there is no reason for the jet to affect significantly the vortex core structure and size.

Figure 10b also confirms the high Reynolds number behaviour of the configurations considered, since an asymptotic evolution of the vortex core size is reached. Indeed, the vortex core size grows only by viscous diffusion (negligible at large enough Reynolds number) once the roll-up is complete. The end of the roll-up phase is approximately reached (also considering the numerical resolutions) at \( \tau \simeq 0.25 \) (i.e. \( x/b \simeq 7.6 \)).

As done in [4], the time-evolution of the vortex intensity is investigated using \( \Gamma_{5-15} \):

\[
\Gamma_{5-15} \triangleq \frac{1}{b/6} \int_{b/12}^{b/4} \Gamma(r) \, dr
\]  

Its time-evolution is presented in figure 11 for the two configurations. The results obtained with the two vortices (positive and negative) are presented since they differ slightly (which was not the case concerning, for example, the effective vortex core size). The first observation thus concerns the dissymmetry of the wake flow which occurs relatively rapidly (from
\( \tau \simeq 0.02 \). The difference between the configurations with and without jets is of the same order of magnitude as that between the positive and negative vortices. Thus, although the vortex intensity is globally slightly (note the vertical scale of the graph) larger with the jets, one concludes that the effect of the jets in a cruise configuration (large initial jet-vortex separation distance) is negligible.

The same conclusion is made concerning the vorticity peaks within the vortices (not shown). The three configurations are very similar and the difference between the configurations with and without jets is of the same order of magnitude as that between the positive and negative vortex due to the dissymmetry of the flow-field. Previous investigations reported a decrease of these vorticity peaks in presence of jets, but in the present case, the large initial jet/vortex separation distance prevents from this effect.

Finally, the vortex structure is investigated by analyzing the circulation and tangential velocity profiles (figure 12), as well as the axial velocity profile (figure 13) at different times. Figure 12a and figure 13a show the profiles at \( \tau = 0.07 \) for the three configurations: propelled wing wake, wing wake with the same high numerical resolution and wing wake with \( h/b = 1/200 \). The same effect of the resolution, as already mentioned, is observed. The configurations with and without jets simulated with the same high resolution are similar in the vortex core. At this early time, the velocity deficit maximum does not yet corresponds to the vortex center (vorticity maximum), confirming the qualitative observations already made (figure 5). Outside the vortex core, the axial velocity profiles of the two configurations obviously differ due to the presence of the jet. The circulation and tangential velocity profiles, however, are strongly similar even outside the vortex core. At later times, the circulation profile is affected by the secondary vortical structures surrounding the vortex cores (figure 12[b, c]). The decrease of the circulation is slightly “displaced” outward in the configurations with jets. But the effect of the jets is still not significant. Concerning the tangential velocity profile, the decay of the peak is significantly more important in the cases without jets. However, this effect is not observed at early times in the wing wake with \( h/b = 1/400 \). It is thus most probably partially induced by the numerical resolution difference. However at later times, the difference is quite significant and thus may also be somehow enhanced by the presence of the jets. As the roll-up is being completed, the velocity deficit maximum decays slowly and progressively corresponds to the vortex center. The jet velocity is wrapped closer to the vortex center. Note however that these profiles are azimuthally averaged, thus considered as axisymmetric. Yet it is not the case outside the vortex core; hence, the low jet velocity obtained.
Conclusions

The effect of two jets on the wake development of a complete propelled wing in cruise configuration was investigated numerically by means of temporal LES. A review of the actual knowledge concerning jet/vortex interactions was performed.

In the present work, the VIC-PFM method (combination of Lagrangian and finite difference methods), was used to carry out the simulation of the time-evolving wing wake. The initial condition corresponds to a wing wake with velocity deficit, developed and analyzed in details in [4], with two added jets. The position of the jets corresponds as much as possible to a realistic cruise configuration, hence, a relatively large initial jet-vortex distance. The strength of the jets is determined so as to compensate the total drag of the wing, in order to correspond to a cruise configuration at constant speed. The latter is the sum of the induced drag, evaluated using the cross-flow kinetic energy, and of the profile drag, obtained from the velocity deficit. This propelled wing wake is compared to the case without jets investigated in [4]. Since the presence of the jets imposes a significantly higher numerical resolution compared to the latter, the simulation of the configuration without jets was been performed with the same high numerical resolution. The comparison of the two configurations without jets thus enables to determine the influence of the numerical resolution on the results.

The global dynamics of the propelled configuration is shown to be very similar to the case without jets. The same instabilities as observed in [4] develop. This strong similarity is explained by the large initial jet/vortex distance and is in agreement with previous results. Still, the jet fluid is entrained by the vortices and is subsequently wrapped around the vortex core. However, it does not penetrate into it. Indeed, when the jet fluid reaches the core periphery, its axial velocity excess has significantly decayed. It finally mixes with the small scale structures generated by the velocity deficit instabilities. During the early development of the wake and jets, the jet region is characterized by the generation of a larger quantity of smaller scales structures. At the end of the roll-up phase, the global flow-field, as well as the vortex structure, are also very similar to those obtained without jets: the two tip-vortices are deformed helically and surrounded by small-scale vortical structures of significant intensity.

The comparison, during the early stages of the wake development, of the two configurations without jets was also performed. Despite the high numerical resolution of both cases, the higher one still provides additional insights in the vortex structure compared to the coarser one. It shows the relevance to perform such challenging numerical simulations when investigating aircraft wake vortices. In the present case, such a high numerical resolution is required for the jets. The computational cost of such a simple configuration, yet as realistic as possible, points out that further developments of our computer capabilities are required before an even more realistic and more complex configuration can be simulated: e.g., the case of a spatially-developing wake with jets, thus requiring a longitudinal computational extend of roughly 10b (instead of the present 0.5b used in the time-developing simulation). Such a simulation is within reach if we run the VIC-PFM code on one thousand processors...
Figure 1: Initial condition: (a) Axial vorticity, $\omega_x t_0$; (b) Axial velocity, $(U - U_\infty)/U_\infty$; (c) transverse vorticity norm, $\omega_{yz} t_0 = \sqrt{\omega_y^2 + \omega_z^2} t_0$. 
Figure 2: Isocontours of vorticity norm ($|\omega| t_0 = 800$ (high opacity) and $|\omega| t_0 = 200$ (low opacity)) colored by the axial component of velocity, at $\tau = 0.04$ for: (a) the propelled wing wake and (b) the wing wake (same high numerical resolution, $h/b = 1/400$).
Figure 3: Longitudinally averaged axial vorticity field, $\overline{\omega_x t_0}$, at $\tau = 0.08$: (a) propelled wing wake; (b) wing wake (same high numerical resolution, $h/b = 1/400$); (c) wing wake ($h/b = 1/200$).
Figure 4: Longitudinally averaged transverse vorticity field, \( \overline{\omega_{yz}} t_0 = \sqrt{\omega_{y}^2 + \omega_{z}^2} t_0 \), at \( \tau = 0.08 \): (a) propelled wing wake; (b) wing wake (same high numerical resolution, \( h/b = 1/400 \)); (c) wing wake (\( h/b = 1/200 \)).
Figure 5: Longitudinally averaged axial velocity field, \((U - U_{\infty})/U_{\infty}\), at \(\tau = 0.08\): (a) propelled wing wake; (b) wing wake (same high numerical resolution, \(h/b = 1/400\)) ; (c) wing wake (\(h/b = 1/200\)).
Figure 6: Isocontours of vorticity norm ($|\omega| t_0 = 800$ (high opacity) and $|\omega| t_0 = 200$ (low opacity)) colored by the axial component of velocity, at $\tau = 0.08$ for (a) the propelled wing wake and (b) the wing wake (same high numerical resolution, $h/b = 1/400$).
Figure 7: Longitudinally averaged axial velocity field, \((U - U_\infty)/U_\infty\), for the propelled wing wake configuration: (a) \(\tau = 0.15\); (b) \(\tau = 0.25\); (c) \(\tau = 0.50\).
Figure 8: Longitudinally averaged axial vorticity field, $\omega_x t_0$, for the propelled wing wake configuration: (a) $\tau = 0.15$; (b) $\tau = 0.25$; (c) $\tau = 0.50$. 
Figure 9: Isocontours of vorticity norm \(|\omega| t_0 = 800\) (high opacity) and \(|\omega| t_0 = 200\) (low opacity) colored by the axial component of velocity, for the propelled wing wake at \(\tau = 0.75\).
Figure 10: Time evolution of (a) the longitudinally averaged vortex core positions and (b) the longitudinally averaged effective vortex core radius: wing wake (dash) and propelled wing wake (solid) at $Re_{\Gamma} = 10^6$.

Figure 11: Time evolution of the longitudinally averaged normalized circulations, $\Gamma_{5-15}/\Gamma_0$, of the positive (thick) and negative (thin) vortex: wing wake (dash) and propelled wing wake (solid) at $Re_{\Gamma} = 10^6$. 
Figure 12: Longitudinally and azimuthally averaged profiles of circulation, $\Gamma(r)/\Gamma_0$ (left), and tangential velocity, $U_\theta(r)/V_0$ (right): propelled wing wake (solid) and wing wake with same high numerical resolution ($h/b = 1/400$) (○) and with $h/b = 1/200$ (dash) at $Re_\Gamma = 10^6$ : (a) $\tau = 0.07$ ; (b) $\tau = 0.2$ ; (c) $\tau = 0.5$. 
Figure 13: Longitudinally and azimuthally averaged axial velocity profiles, \( \frac{U - U_{\infty}}{V_0} \): propelled wing wake (solid) and wing wake with same high numerical resolution \((h/b = 1/400)\) (circles) and with \(h/b = 1/200\) (dash) at \(Re_T = 10^6\): (a) \(\tau = 0.07\); (b) \(\tau = 0.2\); (c) \(\tau = 0.5\).
References


