Roll-up of a temporally-evolving wing wake with velocity deficit

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Roll-up of a temporally-evolving wing wake with velocity deficit
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Abstract

The temporal evolution of a wing wake with initial velocity deficit due to boundary layers is investigated at high Reynolds number by means of Large Eddy Simulations (LES). An efficient combination of Vortex-In-Cell and Parallel Fast Multipole methods, the VIC-PFM method, is used. The initial condition consists in a thin vortex sheet that will roll-up into two counter-rotating far wake vortices. The initial streamwise vorticity component is obtained from Prandtl lifting line theory, applied to an elliptic wing. In our wake model, the boundary layers effects are taken into account by adding a velocity deficit inside the vortex sheet, defining transverse vorticity components. The analysis includes the comparison to the vortex wake without velocity deficit, to the velocity deficit alone and to the same configuration at higher Reynolds number. The longitudinally averaged and three-dimensional dynamics are presented and the time-evolution of the wake structure, including the axial velocity component, is analyzed.

Three successive instabilities develop during the roll-up phase. First, the instability of the transverse vorticity component in the central part of the wake (called velocity deficit instability thereafter) grows and generates vortical structures very rapidly, inducing turbulent motion. Away from the central region, the thin vortex sheet rolls-up to form the primary vortices and thus experiences intense stretching. Closely after the velocity deficit instability, a longitudinal instability develops in the external part of the primary vortices. As it is wrapped and stretched, this sinuous vortex sheet generates a series of vortical structures, secondary vortices wrapped around the primary ones. These large scale structures subsequently burst into small scale turbulence as a consequence of the interaction with the primary vortices and of the viscous diffusion. The same vortex sheet instability develops in the region below the primary vortex, probably triggered by the turbulent motion induced by the velocity deficit instability. Finally, the third instability affects the primary vortex cores. The resulting helical deformation is likely a consequence of the combined effects of the strong axial velocity component into the core, and of the interaction with the secondary vortices. By the end of the roll-up, the secondary vortical structures have significantly decayed, resulting in two wake vortices deformed helically and surrounded by weak turbulence. These mechanisms most likely contribute to the meandering of trailing vortices. At higher Reynolds number, the same dynamics occur. The instabilities develop more rapidly and the secondary vortical structures remain for a longer time due to weaker viscous diffusion.
Introduction

The dynamics of wake vortices have been extensively studied in the past. Any lifting body experiences lift induced drag and generates a wake with two counter-rotating vortices. The intensity of these trailing vortices scales with the lift and the inverse of the velocity. Since the seventies, and the development of large aircraft, aeronautics has been one of the main application of wake vortex investigations. Since the main concern about wake vortices is the potential hazard for following aircraft, research originally focussed especially on the far-field (from several wingspans behind the generating aircraft, where the wake consists essentially in two intense counter-rotating vortices). However, the need to gain a complete understanding of the trailing vortex structure, as well as the interest for industrial problems such as blade/vortex interactions on helicopter blade and propeller cavitation, subsequently led to a large research effort dedicated to the near-field wing tip vortices (including flow field around the wing tip and roll-up). A short review of previous work and present knowledge is presented in Section 1.

The present work was carried out in the framework of the European project FAR-Wake dedicated to challenging aspects of aircraft wake phenomena. The objective of this task was to investigate the effect of the velocity deficit (introduced into the wake due to the boundary layers developing on the wing surfaces) on the roll-up of the vortical wake and on the resulting far field two-vortex system.

The three-dimensional computation of the flow field around a simple, yet realistic, lifting wing together with its near wake remains challenging, especially at the Reynolds number of interest for the aeronautical industry. This may be accomplished in the near future by using efficient numerical methods such as the Vortex-In-Cell Parallel Fast Multipole (VIC-PFM) method used in the present work. For now, it is not the case and the careful computation of wake vortex flows alone is already quite complex and costly.

The approach considered here consists in modelling the wake shed at the trailing edge of a virtual wing in order to simulate its time evolution including the roll-up phase. The well-known lifting line theory, applied to an elliptic wing, is used to obtain the streamwise vorticity component of the initial vortex sheet. A model for the transverse vorticity components induced by the velocity deficit is then also developed to account for the boundary layer effects. The initial condition is described in details in Section 2.2. Temporal Large Eddy Simulations (LES) are carried out using the VIC-PFM method, presented in Section 2.1. The investigation includes the comparison to the corresponding wake without velocity deficit and to the velocity deficit alone. For these three configurations, a relatively high circulation based Reynolds number is considered: \( Re_{\Gamma} = \Gamma_0/\nu = 10^4 \) (\( \Gamma_0 \) being the half plane initial circulation, and \( \nu \) the fluid kinematic viscosity). In order to verify the high Reynolds number behaviour of this configuration, an additional simulation of the same wake with velocity deficit is performed at \( Re_{\Gamma} = 10^6 \). The results and analyses, including the longitudinally averaged and three-dimensional dynamics, the wake structure and the time-evolution of the characteristic quantities, are presented and discussed in details in Section 3. The analysis focusses in particular on the development of instabilities and on the primary vortex structure, including the axial velocity component.
1 Context

In 1998, Spalart performed a general review dedicated to aircraft trailing vortices, including the formation, motion and persistence of wake vortices and relevant to air travel applications [19]. Concerning the formation of tip vortices, the difficulty to model properly the structure resulting from the roll-up phase was highlighted. Some great progress has been made more recently in the modelling of wake vortices after roll-up, using analytically defined induced velocity profiles (2D) [8]. However, as already pointed out by Spalart, an issue in the dynamics of wake vortices, hence in vortex modelling, is the axial velocity component. Indeed, most of the numerical and theoretical investigations neglect this velocity component or use the well-known “q-vortex” which lacks in realism, as stressed in [19] (and references therein).

An interesting (and surprising) conclusion from previous investigations of the tip vortices in the near-field, is that the axial velocity component can be a deficit (directed towards the generating wing) or a surplus (directed downstream the generating wing) [19]. This point is also discussed in [18], in which the near-field behaviour (from the trailing edge to 6.7 chordlengths downstream) of a tip vortex was investigated experimentally in a towing tank. There are two contributions to the axial velocity component in the vortex cores: 1) the boundary layers developing on the wing surfaces, 2) the increase in tangential velocity during the vortex roll-up. The former gives rise to a velocity deficit while the latter induces a “jet-like” axial velocity component. Indeed, the increase of tangential velocity downstream (during the vortex roll-up) gives rise to a favorable axial pressure gradient hence an axial velocity directed downstream. As mentioned in [18], since the axial pressure gradient also depends on the radial position from the vortex center, the distribution of axial velocity in a cross-section of the vortex may exhibits both velocity deficits and surplus. In the towing tank near field measurements of [18], the axial velocity component was dominated by the boundary layer effects (velocity deficit) while in [1], a “jet-like” core axial flow was measured. In the latter, the high level velocity surplus compared to previous investigations was explained by the high angle of attack (10°) and Reynolds number (chord based Reynolds number of 4.610⁶), and by the smoothness of the tip.

Finally, a feature specific to the near-field was the presence of secondary vortices in the periphery of the tip vortex core [18]. Although no clear explanation was given concerning their origin, the effect on the circulation and tangential velocity profiles in the near field was important and diminished as the chord based Reynolds number was increased beyond 10⁵.

Subsequent investigations focussed on the structure of near field wing tip vortices. Among others, the wind tunnel measurements carried out in [9] showed that the flow field was dominated by the wing wake outside the core. Turbulence varied from the central part of the wake to the tip vortex in response to the strain rate imposed by the vortex. In the vortex core, there was no generation of turbulence and the vortex core remained laminar. The presence of secondary vortices was also deduced from velocity measurements. The velocity and circulation profiles suggested a “double-layer” structure with a “classical” inner core growing by viscous diffusion, and an outer core remnant of secondary vortices. This was in agreement with previous investigations ([9] and references therein).

Another point discussed in details in [9], is the wandering (or meandering) phenomenon present in most, if not all, wind tunnel and towing tank investigations. It is characterized
by an unsteady displacement of the vortex core, affecting mean profiles and turbulence measurements. Although this phenomenon is usually attributed to turbulence (induced by the wake of the generating model and/or by the wind tunnel itself), there is no thorough understanding and the origin and physical mechanisms remain an open issue. Recent reviews of the current knowledge and hypothesis concerning the meandering can be found in [12] and [13]. Some more recent hypotheses are: 1) viscous core instabilities developing at large swirl numbers and large Reynolds numbers, and 2) the propagation of Kelvin waves emanating from boundaries. An interesting result concerns the delta wings. Experimental investigations showed that the Kelvin-Helmholtz instabilities developing in the shear layer separated from the leading edge induced high levels fluctuations that likely explained the vortex meandering over delta wings ([13] and references therein). So far, no similar mechanisms have been reported concerning wing tip trailing vortices. The effect of the phenomenon on experimental measurements has been addressed, as in [9] in which a procedure to correct the mean-velocity profiles and to estimate the contribution to the turbulent stresses was developed.

Subsequently to this experimental work, a model of the flow field at the trailing edge of the wing was used to initialize temporal Large Eddy Simulations (LES) of the wake roll-up [22]. One of the two configurations investigated reproduced the experimental conditions of [9]. This study is of great interest for the present work since both have the same approach concerning the initial condition. The model used, derived independently and based on somehow different arguments, appears to be quite similar to the one of the present work (see Section 2.2 for a detailed description). It consisted in a vortex sheet: the streamwise vorticity component was determined from Prandtl lifting line theory, the spanwise and vertical vorticity components corresponded to the streamwise velocity component added to model the wake momentum deficit. The instabilities of the spanwise vorticity component appeared to be an important mechanism that contributed to the production of turbulence. The streamwise wavelength of the predominant instability mode was found to scale with the local (in the spanwise direction) thickness of the vortex sheet, leading to instability waves of different wavelengths and growth rates. Due to the stretching imposed by the vortex in the spiral region (vorticity situated below the vortex core and which is wrapped around it), the instability was amplified, resulting in the formation of vortical structures wrapped around the primary vortices and distributed along the axial direction. These large scale structures were found to cause the undulation of the trailing vortex cores. This phenomenon raised the hypothesis that this mechanism can be related to the wandering (or meandering) of trailing vortices, as previously mentioned in [22] and [9]. They pointed out that this hypothesis was consistent with the observations concerning the meandering phenomenon: amplitude of the core displacement increasing downstream with a preferred direction.

Besides the wandering phenomenon discussed above, the particular conditions of most of the experiments dedicated to near-field wake vortices are worth being pointed out. The first point is that half wings (fixed to a wall) are usually considered in wind tunnel experiments [9] [5] [1]. In [18], the half wing was fixed to an axisymmetric body to be towed into the tank. In such configurations, the boundary condition at the symmetry plane is significantly different from the one of a complete flying wing. Indeed, the wall imposes a no-slip condition. The span loading inevitably decreases to zero there, also affecting somewhat the vortex sheet at the trailing edge of the wing. Also, low aspect ratio wings (half-wings) are considered in these experiments (Ar from 0.603 to 1.5), except in [9] (Ar
= 8.66). This low aspect ratio combined with the use of half wings implies that the region of interest (the tip vortex spiral) is likely significantly affected by the closely no-slip condition of the symmetry plane. The second point is mentioned in the investigation of [1], in which the choice was made to have a large model compared to the test section dimensions. Although the viscous influence of the tunnel was avoided, the inviscid one was known to be quite large. Thus, these particular conditions have to be taken into account when comparing the different observations and conclusions. It also shows the interest of a careful numerical investigation of a somehow realistic wing wake rolling-up in an open domain.

The present work is a contribution to the European project FAR-Wake, started in 2005, with the objective of gaining lacking fundamental knowledge on aircraft wake vortex dynamics. The project focusses on three specific phenomena that need to be further addressed and understood despite the extensive work performed over the past decades: the development of instabilities, the interaction with jets and wakes, and the dynamics In Ground Effect (IGE).

The work presented was carried out in the framework of the Work Package 2 of the project, focussed on the effects of engine jets and fuselage/wing wakes. A brief overview of previous work and present knowledge, as well as a description of the work planned within FAR-Wake WP2, was performed in [16] as a starting point. The present work is a contribution to the Subtask 2.2.2-3 dedicated to the effects of wakes generated by wing elements. The objective is to carry out Large Eddy Simulations (LES) of the roll-up of a temporally-evolving wing wake (longitudinally periodic) with a velocity deficit. The approach, very similar to that used in [22], consists in modelling the vorticity field shed at the trailing edge of a wing by using the lifting line theory. A model is then developed to describe the vorticity components induced by a realistic velocity deficit due to boundary layers, to be added to the wake model. The advantage of this approach is that it takes the roll-up phase into account: no model is used for the trailing vortices which result from the roll-up phase. The effect of the velocity deficit on both the roll-up dynamics and the resulting vortex system can be investigated. Another possibility would be to carry out the simulation of a spatially-evolving wake. This is possible with the numerical method used (described in the next section): it was already successfully applied for ground effect [6] and end-effect [14] investigations. Such a computation of the present configuration would be doable with the computational resources available, yet it would still be costly in time and resources. Moreover, the spatially evolving simulation of an initially inviscid vortex sheet (from lifting line theory, without initial deficit) [14] showed that the true 3D roll-up induced both a velocity deficit (directed towards the generating wing) and a velocity surplus (directed downstream) in the vortex core region. Both had the same order of magnitude: \( U_x \simeq 0.5 V_0 \) \( (V_0 = \Gamma_0/(2\pi b_0)) \), being the characteristic descent velocity of the vortex pair with \( b_0 \) the vortex spacing. Since the derivation of the model for the velocity deficit due to boundary layers (see Section 2.2) leads to an initial velocity deficit approximately 80 times larger, the axial velocities induced by the 3D space-developing roll-up can thus be neglected. Thus, a time-evolving simulation appears to be relevant in the present case.
2 Method

2.1 Three-dimensional combination of Vortex-in-Cell and Parallel Fast Multipole methods

Vortex methods are lagrangian methods and are therefore efficient to simulate incompressible unsteady flows (we refer the interested reader to the reviews in [4, 20, 21]): they have negligible dispersion error and good energy conservation. In the present work, three-dimensional temporal (longitudinally periodic) Large Eddy Simulations (LES) were performed. A hybrid approach, combination of Lagrangian and finite difference methods, is used: the Vortex-In-Cell Parallel Fast Multipole method (VIC-PFM code).

In this new combination of Vortex-In-Cell and Parallel Fast Multipole methods, presented in details in [3], the vorticity form of the incompressible Navier-Stokes equations is solved:

\[
\frac{D}{Dt} \omega = \nabla \cdot (u \omega) + \nabla^2 \omega + \nabla \cdot (\nu_{sgs} (\nabla \omega^s + (\nabla \omega^s)^T)),
\]

where \( \nu_{sgs} \) is the effective subgrid-scale viscosity and \( \omega^s \) the “small-scale” part of the LES field. The numerical solution of equation (1) is sought in a two-fold manner. The convective part is evaluated using a Lagrangian approach, thus with negligible dispersion errors. The time variation of the vortex particle strength, that includes both the vortex stretching and the dissipation terms, is solved on a regular grid, using 2nd order finite differences. Interpolation from the Lagrangian particles to the Eulerian grid, and back, is done using the \( M'_4 \) scheme [4, 21]. Once on the Eulerian grid, the stream-vector \( \psi \) is evaluated by solving the Poisson equation

\[
\nabla^2 \psi = -\omega.
\]

The velocity field, needed for convection and stretching, is then obtained by evaluating \( u = \nabla \times \psi \) using, again, 2nd order finite differences. The global time marching procedure is carried out using the 2nd order Leap Frog scheme for the convection and the 2nd order Adams-Bashford scheme for the rest. Finally, the divergence-free character of the vorticity vector field is maintained by a proper reprojection of the discrete vorticity field (which also requires solving a Poisson equation).

Particular to the present implementation of this procedure is the treatment of the boundary conditions on the grid when solving equation (2). At a given time step, the boundary conditions for \( \psi \) are determined using the Green’s function approach via a Parallel Fast Multipole method (PFM). With this approach, the unbounded domain condition can be ensured accurately on a relatively small grid: only as large as the vorticity field itself. This allows for a significant reduction of the computational cost for solving equation (2) when compared with more classical methodologies [20, 3, 21]. Furthermore, the method can be parallelized using a domain decomposition approach: the PFM code, which has a global view of the whole field, is then used to obtain proper boundary conditions on each subdomain [20, 3, 21], leading to a very efficient numerical tool.

The LES modelling is here done using a multiscale subgrid-scale model: the model solely acts on the small scale part of LES field, \( \omega^s \). It is obtained from the complete LES field, \( \omega \), using a compact discrete filter (applied iteratively). We here use our version of the Regularized Variational Multiscale (RVM) model [2]. The advantage of such model is that it preserves the inertial range while providing dissipation at the high wave numbers:
it is only active during the complex phases of the flow, while remaining inactive in the
laminar and/or well-resolved regions.

As already mentioned above, the temporal evolution of the wake is investigated in the
present work. Temporal LES are therefore carried out using periodic boundary conditions
in the longitudinal direction. The length of the computational domain is \( L_x / b = 0.5 \), where
\( b \) is the wingspan of the wing. In the transverse directions, open domain conditions are
used. The initial conditions, described in details in the next section, are two-dimensional
vorticity fields extruded in the periodic direction. No perturbation was added. The
simulations were performed on 8 Opteron processors at 2.6 Ghz. The time step was
adapted during the simulation in order to capture the large flow variations occurring in
the early stages of the thin vortex sheet roll-up, and to optimize the computational time
subsequently. Thus, the time steps ranged from \( \Delta t / t_0 = 5 \times 10^{-5} \) to 2.5 \( 10^{-4} \) (where
\( t_0 = b_0 / V_0 \) is the characteristic time of the vortex pair defined in Section 3). The initial
number of particles was approximately \( 4 \times 10^5 \) for the reference case and for the cases with
velocity deficit (described below). By the end of the simulation, the number of particles
was about \( 7 \times 10^5 \), and the Vortex-In-Cell grid had \( 14 \times 10^6 \) grid points.

2.2 Initial conditions: description of the wake model

As already mentioned, the approach of the present work consists in modelling the vorticity
field shed at the trailing edge of a generating wing. The objective is to investigate the
effect of the axial velocity deficit due to boundary layers on the roll-up phase and on the
resulting vortex system. In order to investigate the dynamics in details, two additional
configurations are compared to the one of interest: 1) a wake roll-up without velocity
deficit is used as the reference case and 2) a velocity deficit without wake vortex is used
to “isolate” the dynamics of the velocity deficit. The wake with velocity deficit then
corresponds to the combination of these two configurations.

Concerning the vortex wake configuration, only the axial vorticity component is ini-
tially non-zero. The lifting line (inviscid) theory is applied to obtain the initial vortex
sheet shed by an elliptically loaded wing (elliptic chord and lift distributions along the
span). The theory consists in replacing the lifting wing of lift distribution, \( l(y) \), by a
vortex line defined by its span loading, \( \Gamma(y) \), using the relation: \( l(y) = \rho U_\infty \Gamma(y) \) (\( \rho \) being
the fluid density and \( U_\infty \) the flight speed). For an elliptically loaded wing, one obtains:

\[
\Gamma(y) = \Gamma_0 \sqrt{1 - \left( \frac{y}{b/2} \right)^2} \tag{3}
\]

where \( \Gamma_0 \) is the half plane total circulation. It characterizes the wake together with the
vortex spacing (after roll-up), \( b_0 \):

\[
\int_{-b/2}^{b/2} \Gamma(y) dy = \Gamma_0 b_0 \tag{4}
\]

with \( b_0 = \frac{\pi}{4} b \) for elliptical loading. Considering the definition of the total wing lift, and
the lift coefficient, respectively:

\[
L = \int_{-b/2}^{b/2} l(y) dy = \rho U_\infty \int_{-b/2}^{b/2} \Gamma(y) dy = \rho U_\infty \Gamma_0 b_0 \tag{5}
\]

\[
C_L = \frac{L}{\frac{1}{2} \rho U_\infty^2 S} = \frac{2 \Gamma_0 b_0}{U_\infty S} \tag{6}
\]
with \( S = \frac{b^2}{Ar} \) the wing surface \((Ar\) being the wing aspect ratio), the characteristics of the wake can be expressed as a function of the wing characteristics as follows.

\[
\Gamma_0 b_0 = \frac{1}{2} U_\infty b^2 \frac{C_L}{Ar} \tag{7}
\]

One also obtains

\[
V_0 = \frac{\Gamma_0}{2 \pi b_0} = \frac{U_\infty}{4 \pi} \left( \frac{b}{b_0} \right)^2 \frac{C_L}{Ar} \tag{8}
\]

and thus, for an elliptical loading,

\[
\frac{V_0}{U_\infty} = \frac{4}{\pi^3} \frac{C_L}{Ar} \tag{9}
\]

In the present work, the wing characteristics are the following:

\[
C_L = 1.5 \tag{10}
\]

\[
Ar = 7.5 \tag{11}
\]

thus, \( \frac{C_L}{Ar} = 0.20 \) and

\[
\frac{V_0}{U_\infty} = 2.58 \times 10^{-2} \tag{12}
\]

The Reynolds number is taken as:

\[
Re_\Gamma = \frac{\Gamma_0}{\nu} = 10^4. \tag{13}
\]

The corresponding shed vorticity field is a vortex sheet with circulation per unit length:

\[
\gamma(y) = -\frac{d\Gamma}{dy}(y) \tag{14}
\]

This singular vortex sheet is then regularized using a gaussian kernel in order to produce a regular vorticity field:

\[
\omega_\sigma(y, z) = \frac{1}{\pi \sigma^2} \int_{-b/2}^{b/2} \exp \left( -\frac{(y - y')^2 + z^2}{\sigma^2} \right) \gamma(y') dy' \tag{15}
\]

where \( \sigma \) is the regularization parameter, set in the present work to \( \sigma/b = 1/75 \), still giving a fairly thin vortex sheet. The grid resolution is \( h/b = 1/200 \).

The resulting vorticity field is presented in figure 1. It corresponds to the initial condition of the reference case to be compared to the case with velocity deficit. Since the velocity deficit is a model for the boundary layers developed along the chord of the wing surface, the same elliptic distribution (as for the chord and lift) is here used:

\[
U_w(y, 0) = U_{w0} \sqrt{1 - \left( \frac{y}{b/2} \right)^2}
\]

\( U_w = U - U_\infty \) is a velocity deficit, with \( U \) the axial velocity component and \( U_\infty \) the freestream velocity. The parameter \( U_{w0} \) determines the magnitude of the velocity deficit. The axial direction, \( x \), is defined positively downstream of the wing: the velocity deficit,
$U_w$, is thus negative. It is then regularized to obtain a 2-D axial velocity field. The velocity deficit is thus defined by two parameters: its magnitude, $U_{w0}$, and its thickness (set by the regularization parameter $\sigma$).

The approach used to find realistic parameters consists in considering the different contributions to the total drag. As mentioned in [7] (and reference therein), the induced drag contributes to 40 to 50% of the total drag in cruise. Since the induced drag is well approximated by the cross-flow kinetic energy in a close plane behind the wing ([8] and references therein), it is obtained by computing the cross-flow kinetic energy per unit length of the initial vorticity field described above:

$$D_i \simeq E_{c0} = \frac{1}{2} \rho \int_A (v^2 + w^2) dA = \frac{1}{2} \rho \int_A \psi_x \omega_x dA$$

(16)

where $v$ and $w$ are the velocity components in $y$- and $z$-direction, $A$ is the cross plane (orthogonal to the flight direction $x$), $\omega_x$ is the $x$-component of vorticity and $\psi_x$ is the stream-function satisfying the Poisson equation, $\nabla^2 \psi_x = \frac{\partial^2 \psi_x}{\partial y^2} + \frac{\partial^2 \psi_x}{\partial z^2} = -\omega_x$.

The total drag is the sum of the induced drag and of the “profile” (viscous) drag. In the configuration investigated here, the velocity deficit is a direct consequence of the development of the boundary layers on the airfoil. The velocity deficit must be defined such that the corresponding “profile” drag contributes to 50 to 60% of the total drag. For the spatially-evolving wake, the “profile” drag is obtained as:

$$D_p = \rho \int_A (U_\infty - U) U dA$$

(17)

with $U_\infty$ the freestream velocity and $U$ axial velocity. In the present case, the relative velocity deficit is as $U_w = -(U_\infty - U)$, and thus, the profile drag is obtained as:

$$D_p = -\rho \int_A (U_\infty + U_w) U_w dA$$

(18)

In order to determine $U_{w0}$, the fact that the initial flow field corresponds to the one shed at the trailing edge of the generating wing is considered. Concerning the axial velocity field, the no-slip condition imposes the velocity to decrease to zero at the trailing edge (center line of the initial vortex sheet). Thus, from this point of view, the natural value for $U_{w0}$ is $-U_\infty$, corresponding to $U = 0$.

Concerning the regularization parameter: the same as for the vorticity field is used to obtain the axial velocity field. Thus, the boundary layers have roughly the same thickness as the vortex sheet shed by the wing (from lifting line theory). This set of parameters leads to an induced drag, $D_i$, and a profile drag, $D_p$, contributing respectively to 42.4% and 57.6% of the total drag, which is in fair agreement with the respective contributions in cruise.

The initial velocity profiles, and the corresponding vorticity and velocity fields obtained are presented in figures 2 and 3, respectively. The initial condition presented in figure 3 corresponds to the velocity deficit configuration. The combination of the initial conditions presented in figures 1 and 3 corresponds to the wake with velocity deficit.

The results of a fourth simulation, similar to the wake with velocity deficit but at higher Reynolds number, $Re_\Gamma = 10^6$, are also presented for comparison. The objective is to
investigate the effect of the Reynolds number on the roll-up and to show that the simulation is indeed representative of a high Reynolds number wake. However, it is important to stress that it is different from an experimental investigation of the Reynolds number effect, in which the flow over the wing (the boundary layers thickness, hence, the velocity deficit) would be different. In the present case, the velocity deficit presented above is the same for the two different Reynolds numbers. Table 1 sums up the configurations simulated and discussed in the next sections. The kinetic energy per unit length of the axial velocity component, defined below, is also given.

\[ E_d = \frac{1}{2} \rho \int_A U_w^2 \, dA \quad (19) \]

### Table 1: Summary of the initial conditions of the four configurations investigated.

<table>
<thead>
<tr>
<th>Configuration</th>
<th>( \frac{U_{\infty}}{U_\infty} )</th>
<th>( Re )</th>
<th>( \frac{D_r \rho}{\Gamma_0} )</th>
<th>( \frac{E_{d0}}{\rho \Gamma_0^2} )</th>
<th>( \frac{D \rho}{\Gamma_0^2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reference (Ref)</td>
<td>0</td>
<td>10^4</td>
<td>0.364</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Velocity Deficit (Def)</td>
<td>-1</td>
<td>same</td>
<td>( \nu )</td>
<td>0</td>
<td>0.343</td>
</tr>
<tr>
<td>Ref + Def (RUDef)</td>
<td>-1</td>
<td>10^4</td>
<td>0.364</td>
<td>0.343</td>
<td>0.495</td>
</tr>
<tr>
<td>High ( Re ) RUDef</td>
<td>-1</td>
<td>10^6</td>
<td>0.364</td>
<td>0.343</td>
<td>0.495</td>
</tr>
</tbody>
</table>

#### 3 Results

In this section, the results obtained for the four configurations described above are presented and compared. For the time-evolution, the non-dimensional time is used:

\[ \tau = \frac{t}{t_0} \text{ with } t_0 = \frac{b_0}{V_0} = \frac{2 \pi b_0^2}{\Gamma_0} \]

The corresponding downstream position of the wake, \( \frac{x}{b} \), is obtained from \( x = t U_\infty \). This gives

\[ \frac{x}{b} = \left( \frac{\pi}{2} \right)^2 \frac{Ar}{C_L} \tau = 30.4 \tau \]

First, the qualitative dynamics (two- and three-dimensional) are presented. Then, a detailed investigation of the time-evolution of the vortex system is carried out. Velocity and circulation profiles, as well as different diagnostics, are analyzed.

#### 3.1 Qualitative dynamics

##### 3.1.1 Longitudinally averaged dynamics

First, the longitudinally averaged fields are discussed. Figure 4 presents the time-evolution of the axial vorticity field, \( \omega_x \), for the reference case. The “classical” roll-up of the vortex sheet is observed. The vorticity, initially distributed along the span rolls-up into two counter-rotating vortices to be mainly concentrated into the vortex cores after roll-up. The dynamics remain two-dimensional during the complete roll-up, leading to two longitudinally uniform counter-rotating vortices. The spanwise and vertical vorticity components,
$\omega_y$ and $\omega_z$, respectively, are initially zero and remain negligible. Due to the longitudinal uniformity, no axial velocity is generated.

These dynamics are to be compared to the case with added velocity deficit presented figure 5. An instability developing very early (already well developed at $\tau = 0.01$, figure 5a) in the central part of the thin vortex sheet is clearly observed. Figure 6 shows the time evolution of the transverse vorticity norm, $\omega_{yz} = (\omega_y^2 + \omega_z^2)^{1/2}$ and of the axial velocity for the velocity deficit alone configuration. The thin spanwise vortex sheet, predominant in the central part of the wake, appears to be highly unstable. The same feature is observed in the central part of the wake with added velocity deficit on figure 7a. This comparison confirms that the instability is initiated due to the spanwise vorticity component concentrated into the thin sheet. The time-evolution of the flow field, for the velocity deficit alone, shows that this instability growths very rapidly from $\tau = 0.02$ to $\tau = 0.05$, generating vortical structures and inducing turbulent motions. From $\tau = 0.05$, the longitudinally averaged vorticity field exhibits a turbulent behaviour in its central part (figure 6).

The same dynamics occur in the central part of the wake with added deficit (figure 7a and 7b). As was observed in figure 5, the instability of the spanwise vorticity sheet induces a characteristic feature concerning the axial vorticity component: “patches” of high axial vorticity magnitude, alternatively positive and negative, are distributed along the span in the central part. Away from it, near the rolling-up vortex, the vorticity sheet is wrapped (and stretched) around the vortex core and remains coherent (figure 7b). Opposite-signed axial vorticity is also generated in this region. Subsequently, the instability of the central part of the wake spreads towards the extremities of the sheet. At $\tau = 0.1$, vortical structures resulting from this instability are surrounding the two primary vortex cores (figures 5c and 7c). Finally, figures 5d and 7d show that, subsequently, the flow field consists essentially in two coherent vortices surrounded by weak small scale turbulent vortical structures.

These observations reveal that the velocity deficit (and corresponding vorticity components) causes the initially two-dimensional flow field to rapidly become three-dimensional. The three-dimensional topology is presented and discussed in the next section.

### 3.1.2 Three-dimensional dynamics

The time evolution of the three-dimensional wake dynamics with velocity deficit is presented in figure 8, using two isocontours of vorticity norm, $|\omega| = (\omega_x^2 + \omega_y^2 + \omega_z^2)^{1/2}$, colored by the axial vorticity component, $\omega_x$. The three-dimensional visualizations of both the reference and velocity deficit cases are not presented. The former remains two-dimensional, so the dynamics are well characterized by the averaged fields presented above. Concerning the latter, the dynamics are essentially similar to the ones observed in the central part of the wake with deficit discussed below.

Figure 8 shows the early roll-up during which the instability develops in the central part of the wake. The vortex sheet exhibits short wavelength instabilities in the spanwise direction (corresponding to the instability of the spanwise vorticity component observed on figure 7a), with “vortex tubes” of alternatively positive and negative axial vorticity. The flow field in this region is somewhat particular but the region of interest is slightly away from the central part, in the region which is wrapped around the primary vortex.
At $\tau = 0.04$, one can observe the initiation of a longitudinal instability in the thin vortex sheet wrapped around the external part of the right hand side (rhs) vortex. Figure 8 also shows the subsequent steps of the dynamics. The instability grows rapidly and the perturbed vortex sheet surrounds the external half part of the vortex. Also, similar perturbations start developing in the region below the vortex. This feature is observable on the lhs tip vortex and is enhanced at $\tau = 0.06$. In this region, the instability is likely to be triggered by the turbulence induced by the early velocity deficit instability that spreads towards the primary vortices. At this time, the instability in the external periphery of the primary vortex results in secondary vortices wrapped around it. These observations are in good agreement with the investigation in [22] (discussed in Section 1), in which similar vortical structures, called “ribs” were observed. The wavelength and growth rate of the so-called “sinuous modes” were found to vary along the spanwise direction due to the variation of the vortex sheet thickness. This is in agreement with the present observations. The instability first develops in the external periphery of the primary vortex, where the stretching is high and the vortex sheet is thin (high growth rate). Subsequently, it develops in the region situated between the primary vortex and the central part of the wake (probably triggered by the turbulent motion of the central part of the wake), which is entrained by the primary vortex but is thicker and has experienced weaker stretching so far (lower growth rate). The difference in the wavelength is observable in figure 8. At $\tau = 0.07$, around the tip vortex, one can observe six wavelengths in the domain length (six secondary vortices) while below the lhs primary vortex, four wavelengths are observed. At $\tau = 0.08$, the secondary vortex structures completely surround the primary vortices.

This feature appears to be similar to that observed in [17] and [15], in which the interaction of a single vortex with a turbulent hot jet was studied by means of LES. Similar vortex structures were wrapped around the primary vortex in the case of an initial short jet to vortex separation distance (but not in blowing jet configurations). Similarly to the present case, this feature was the consequence of the intense stretching induced by the roll-up of the jet around the vortex. A physical explanation for the emergence of these three-dimensional structures, based on the azimuthal vorticity equation, is given in [17]. This comparison shows the somewhat independence of the two instabilities observed in the central part of the wake with velocity deficit and the one developing in the vicinity of the tip vortex. Indeed, in [15], the initially axisymmetric jet did not exhibit any instability before being rolled-up (and intensively stretched) around the primary vortex. In the roll-up of the reference case (without velocity deficit), the same intense stretching is experienced by the thin vortex sheet (axial vorticity only) as it rolls-up into the primary vortex. Recalling that no instability develops in the reference case points out that the origin of the instability is the stretching of the transverse components of vorticity, $\omega_y$ and $\omega_z$ (induced by the axial velocity deficit).

Already, at $\tau = 0.08$, the deformation of the primary vortex core due to the interaction with the secondary vortices is initiated. Subsequently, the wrapping and intense stretching induced by the primary vortices on the surrounding secondary vortices result in the bursting of the latter into small scales vortical structures. In the same time, the deformation of the primary vortex core is enhanced, resulting in a characteristic helical deformation. This phenomenon is of great interest and is likely related to the well known meandering effect observed in wind-tunnel experiments of trailing vortices (Section 1).

At $\tau = 0.25$, the roll-up of the wake is essentially complete (this point is discussed in more details in Section 3.2). The wake after roll-up thus consists in two vortices exhibiting
helically deformed vortex cores surrounded by small scales turbulent vortical structures. This surrounding turbulence then decays through viscous diffusion.

For the configuration at \( Re_G = 10^6 \), the same three-dimensional dynamics are essentially observed. The consequence of the weaker viscous effects are the even faster development of the instabilities, in particular concerning the instability of the vortex sheet wrapped around the primary vortex. The second difference is the slow decay of the secondary vortical structures compared to the case at lower Reynolds number. These effects clearly appear when comparing figure 8 to figure 10. In the next section, the time-evolution of the wake structure, including the axial velocity distribution, and of characteristic quantities, as well as the instability mechanisms, are analyzed in more details (also quantitatively).

3.2 Quantitative analysis

3.2.1 Roll-up phase and resulting vortex structure

The first point of interest is to characterize the global time-evolution of the wake and to determine the time when the roll-up is complete. This is useful to characterize the vortex structure after roll-up that is used as initial condition in most numerical simulations of wake vortices. Usually, the downstream distance after which the wake can be considered developed is taken as 8 to 10 \( b \) (\( b \) being the wingspan).

Figure 11a shows the mean trajectory of the primary vortex centers (defined as the locations of maximum and minimum longitudinally averaged axial vorticity in the tip regions) for the three cases: reference at \( Re_G = 10^4 \), wake with velocity deficit at \( Re_G = 10^4 \) and wake with velocity deficit at \( Re_G = 10^6 \) (run until \( \tau = 0.75 \) for the latter). One criterion that may be used to define the end of the roll-up phase is to consider the time when the vortex spacing is constant, at \( b_0 = \frac{\pi}{4} b \). For the reference case, it occurs between \( \tau \approx 0.20 \) and \( \tau \approx 0.25 \) (\( z/b \approx -0.167 \) on the graph 11a). For the cases with deficit, the trajectory is perturbed by the vortical structures and turbulence surrounding the primary vortices (as described in the previous section). However the initial roll-up phase matches very well with the reference case, and the same roll-up time can be assumed. This time, from which the vortex spacing remains constant, should correspond to the time at which the vortex strength is maximal. Indeed, during the roll-up phase the tip vortices are fed by the rolling-up vortex sheet. Then, the vortex strength will decay by viscous diffusion. A good estimate of the vortex strength used for aircraft wake vortices (representative of the induced rolling moment) is \( \Gamma_{5-15} \), defined as the circulation averaged between 5 and 15 meters for a wingspan of \( b = 60 \) m. The generalization of \( \Gamma_{5-15} \) to any wingspan reads:

\[
\Gamma_{5-15} \triangleq \frac{1}{b/6} \int_{b/12}^{b/4} \Gamma(r) \, dr
\]  

(20)

Figure 12 shows the time-evolution of \( \Gamma_{5-15} \) for the three cases. The behaviour depicted above is verified by the reference case: it increases during the roll-up, reaches a maximum and decreases slowly by viscous diffusion. The maximum occurs at \( \tau \approx 0.25 \), confirming the roll-up time obtained previously. This time corresponds to a downstream distance of \( x/b = 7.6 \): this is in good agreement with that obtained in spatially-developing simulations [14].
It is now possible to obtain the effective vortex core size, $r_c$, after roll-up, the time-evolution of which is depicted in figure 11b. It is defined as the radial position where the vortex induced velocity, $U_\theta(r) = \Gamma/(2\pi r)$ is maximal. At the end of the roll-up phase, at $\tau = 0.25$, one obtains, for both the reference wake and that with velocity deficit:

$$\frac{r_c}{b} \simeq 0.039$$  \hspace{1cm} (21)

However, notice that this is an averaged value and that the vortex core is already well deformed at this time, in the case with velocity deficit.

Figure 11b also shows the Reynolds number effect on the vortex core size. Although the increase of vortex core radius is lower at higher Reynolds number (due to a weaker viscous diffusion), the same global evolution is exhibited for all three cases until $\tau = 0.35$. For the wake with velocity deficit at $Re_\Gamma = 10^6$, the vortex core radius is $r_c/b \simeq 0.032$ at $\tau = 0.25$. From $\tau = 0.35$, it then remains constant, showing that the viscous diffusion of the vortex core is negligible at this Reynolds number, and that an asymptotic behaviour has been reached. It also suggests that the roll-up time is $\tau = 0.35$ (corresponding to a downstream distance of $x/b = 10.7$) rather than $\tau = 0.25$. At this time, the effective vortex core size, for $Re_\Gamma = 10^6$, is:

$$\frac{r_c}{b} \simeq 0.035$$  \hspace{1cm} (22)

Recall also that the values of $r_c/b$ obtained here correspond to the roll-up of a vortex sheet of Gaussian regularization with $\sigma/b = 1/75$.

Another piece of information provided by the time-evolution of $\Gamma_{5-15}$, concerns the cases with velocity deficit. The initial evolution is similar to the reference case. The first slight deviation occurs at $\tau \simeq 0.01$ corresponding to the generation of opposite signed axial vorticity in the spiral region due to intense stretching, and in the central part of the wake. Then, after $\tau = 0.04$ for $Re_\Gamma = 10^4$ and $\tau = 0.03$ for $Re_\Gamma = 10^6$, the $\Gamma_{5-15}$ decreases as a consequence of the early development of the longitudinal instability of the wrapping vortex sheet and the generation of opposite signed vorticity and turbulence.

The local effects of the surrounding secondary vortical structures and turbulence on the primary vortex structure can be observed on figure 13. The $\Gamma(r)$ circulation and $U_\theta(r)$ tangential velocity profiles are shown at different times. Inside the vortex core, up to $r \simeq r_c$, the circulation and tangential velocity profiles remain very similar for the two cases at $Re_\Gamma = 10^4$. The peak of tangential velocity is however slightly higher for the case with velocity deficit. This may be due to the contribution of the secondary vortical structures surrounding the primary vortex cores. As time goes on, it decays faster compared to the reference case due to the interaction with the vortical structures and surrounding turbulence. Outside the core, from $r \simeq r_c$, the two cases differ and the initial slight deviation is enhanced as the instability generating the secondary vortical structure grows. The circulation and tangential velocity are lower compared to the reference case in this region. Outside this region, from $r \simeq 0.15$ at $\tau \simeq 0.07$ and from $r \simeq 0.3$ at $\tau \simeq 0.5$, the circulation and tangential velocity profiles are very similar in both cases. At $Re_\Gamma = 10^6$, the decay of the peak of tangential velocity is weaker and the effects of the secondary vortical structures are more important.

The time-evolution of the kinetic energies for the reference wake and for the wake with velocity deficit is presented figure 14. In the reference case, the total kinetic energy corresponds to the cross-flow kinetic energy since the flow field remains two-dimensional,
without axial velocity component. It decreases slowly by viscous diffusion. For the cases with velocity deficit, the total two-dimensional kinetic energy is the sum of the cross-flow and axial-flow kinetic energies. It appears that in the present configuration, both are of the same order. The initial kinetic energy for the wake with velocity deficit is thus twice that of the reference case. The drastic effect of the velocity deficit instability is evident: from $\tau = 0$ to $\tau = 0.2$ the axial-flow kinetic energy decreases very rapidly. Then, the curve exhibits a viscous decay similar to that of the cross-flow kinetic energy of the reference case. There is also, at early times, a slight overshoot in the cross-flow kinetic energy, showing that a small amount of the axial-flow kinetic energy has been transferred to the cross-flow. Then, the decay is enhanced compared to the reference case, due to the turbulent interactions of the vortical structures generated by the instabilities (until $\tau = 0.2$). From $\tau = 0.2$, the surrounding vortical structures are essentially dissipated; only weak turbulence remains. The energy then decays similarly to the reference case. At higher Reynolds number, the decay of the axial flow kinetic energy induced by the instabilities is slightly enhanced. Concerning the cross-flow kinetic energy, the same behaviour as in the case at lower Reynolds number is observed. The overshoot is slightly more important at large Reynolds number. Then, the decay is equivalent in both cases, until $\tau \approx 0.4$. The enhanced instability mechanism in the large Reynolds number case thus compensates for the weaker viscous diffusion. From $\tau \approx 0.4$, the surrounding turbulent vortical structures are significantly dissipated and the energy decays slightly slower than in the case at lower Reynolds number, due to weaker viscous diffusion. From a global point of view, the difference at higher Reynolds number is not very significant, showing the high turbulent behaviour of the configurations investigated.

Finally, figure 15 shows the vertical and axial linear impulses. The longitudinally averaged linear impulse is defined by:

$$I = \frac{1}{L_x} \rho \int_V x \times \omega \, dV$$

where $V$ is the periodic computational domain. Thus, the vertical and axial linear impulses are given by:

$$I_z = \frac{1}{L_x} \rho \int_V (x \omega_y - y \omega_x) \, dV = \rho \int_A (x \bar{\omega}_y - y \bar{\omega}_x) \, dA$$

and

$$I_x = \frac{1}{L_x} \rho \int_V (y \omega_z - z \omega_y) \, dV = \rho \int_A (y \bar{\omega}_z - z \bar{\omega}_y) \, dA$$

respectively.

The vertical linear impulse is related to the lift by: $L = -U_\infty I_z$. Figure 15 shows that both the vertical and axial linear impulses are well conserved by the numerical method used, as it should be. Of course, the axial linear impulse for the reference case is zero.

### 3.2.2 Axial velocity component and core structure

So far, the transverse vorticity component resulting from the initial velocity deficit has been seen to cause the development of large growth rate instabilities, affecting significantly the flow field. In this section, we focus on the time-evolution of the axial velocity distribution in the wake. Its importance in the wake vortex dynamics has already been pointed out in Section 1.
Figure 16 presents the time evolution of the longitudinally averaged axial velocity field. Initially, the strong axial velocity deficit is concentrated in the central part of the thin sheet. The effect of the velocity deficit instability is observable. The induced turbulent motion causes the velocity deficit to decay rapidly. As the vortex sheet rolls-up into the two primary vortices, a part of the velocity deficit is entrained into the vortices. In figure 16, the color scale was voluntarily kept constant for all times in order to visualize the decay. Figure 17 shows the axial velocity field at $\tau = 0.25$ with a rescaled color scale. The indication of the vortex core centers shows that the axial velocity deficit is concentrated into the vortex cores, the maximum velocity position matching with the maximum vorticity one. Therefore, it appears relevant to consider the longitudinally and azimuthally averaged axial velocity profile, $U(r) - U_\infty$, depicted figure 18 at different times. It confirms that the velocity deficit is concentrated in the vortex core and that it remains significant despite the observed decay. The “bump” observed at $\tau = 0.07$, at $r/b \approx 0.05$, is characteristic of the presence of secondary vortices whose axes are approximately perpendicular to that of the primary vortex, hence, the contribution to the axial velocity component. Then, the peak of velocity deficit in the core decays, as well as the secondary vortices whose characteristic signature is no longer observed. From this time, the axial velocity profiles appear smoother, hence the longitudinally averaged flow is completely rolled-up and more axisymmetric.

Concerning the comparison of the two Reynolds numbers, the difference is relatively significant only during the roll-up phase in the vortex core, where the peak is higher at higher Reynolds number. At $\tau = 0.5$, the axial velocity profiles are similar. The high Reynolds number case decays slightly faster: at later times (not shown), the maximum axial velocity is a bit lower than that at $Re = 10^4$. This may be due to the interaction with the secondary vortical structures whose strength and life-time are greater at higher Reynolds number (due to the weaker viscous diffusion).

In figure 18, the dimensionless axial velocity is provided, using the vortex pair descent velocity $V_0$, in order to enable the comparison to the maximum tangential velocity (figure 13b). Indeed, this significant axial velocity component is likely to play a role in the instability mechanism of the primary vortex cores, and one would be tempted to analyze the swirl number, $S = U_0(r_c)/(U(r_c) - U_\infty)$, to determine the possibility for q-vortex-like instabilities to develop. However, in the present work, the tangential and axial velocity distributions are far from q-vortex’s gaussian ones. Moreover, in the present work, the vortices not only consists in vortices with strong axial velocity, but also surrounded by large scale structures and turbulence. Thus, the comparison to the theory available for analytical q-vortices would be questionable. Indeed, the deformation of the vortex core is likely the consequence of the combined effects of the strong axial velocity component present in the vortex cores, and of the secondary vortices surrounding the primary ones.

The deformation observed in figure 8 is very similar to helical instabilities. These instabilities are investigated in details in the case of a single Lamb-Oseen vortex in [11]. The helical instabilities are classified into several “families”. For the reasons mentioned above, the precise identification of a relevant “family” is not that obvious. Nevertheless, characteristic axial perturbation fields are exhibited. These fields correspond to the fluctuations of axial vorticity, $\omega_x' = (\omega_x - \overline{\omega}_x)$. Figure 19 shows this perturbation field in the vicinity of the positive tip vortex (counterclockwise rotation), at a particular longitudinal position and for different times. At $\tau = 0.05$, two “lobes” of weak intensity and of opposite signed vorticity are observed into the vortex core. At later times, the intensity of these vorticity
lobes increases. Such a structure is characteristic of some of the “families” of helicoidal modes. A surprising feature is the one presented in figure 20. The same kind of perturbation field (same longitudinal position) is shown but at $\tau = 0.2$, thus corresponding to the structure found between figure 19c and figure 19d. The structure of the perturbation field is different: four lobes in the periphery of the vortex core, the lobes of same vorticity sign facing each other. This structure is similar to the “F” family depicted in [11]. Moreover, the three-dimensional structure exhibited by the corresponding mode is comparable to that observed in figure 8.

An additional feature concerns the dissymmetry of the wake due to the complex three-dimensionality of the flow-field. This is verified in the deformation of the vortex core. The positive vortex (counterclockwise rotation) is more deformed than the negative one (figure 10). Therefore, the perturbation fields of figure 19 are presented for the former. The perturbation fields for the negative vortex (not shown here) exhibit weaker fluctuations, confirming the relation of these perturbation fields to the vortex core helical deformation.

As mentioned above, one cause of these deformations is likely the secondary vortices wrapped around the tip vortices. One can notice that the configuration is quite similar to the one occurring for vortices in ground effect (IGE) when the unstable secondary vortex emanating from the boundary layer wraps around the primary one. Eventually, the primary vortex also exhibits a longitudinal instability ([10] and [6]). However, the core deformation occurring IGE is not helicoidal. Thus, the presence of the secondary vortices is likely to enhance the instability of the tip vortex (like IGE) but the helicoidal structure of the deformation is here attributed to the presence of the strong axial velocity deficit (which was not accounted for in [10] and [6]).
Conclusions

Temporal Large Eddy simulations of the roll-up of an elliptic wing wake with velocity deficit due to boundary layers are presented. The initial condition consists in a streamwise vorticity component obtained from Prandtl lifting line theory, with added transverse vorticity components corresponding to an axial velocity deficit. A reference case without deficit, and a case with velocity deficit alone (without axial vorticity component) are also investigated for comparison. A numerical simulation of the same wake with velocity deficit is also performed at higher Reynolds number to confirm the high Reynolds number behaviour.

The present work is globally in very good agreement with previous investigations (mainly experimental) dedicated to near field wake vortices. The careful numerical investigation, complementary to the existing experimental ones, provides insights into previously reported features as well as new details concerning the complex dynamics of this flow field. The wake roll-up appears to be drastically affected by the presence of the axial velocity deficit (and of the corresponding vorticity components). A detailed investigation of the dynamics is performed, showing the occurrence of three instabilities of somehow different nature. The longitudinally averaged and three-dimensional topologies of the time-evolving flow field are described.

First, an early instability develops very rapidly in the central part of the wake. The origin of this “velocity deficit” instability is the transverse vorticity component. This instability is particularly enhanced in the present case because of the thin velocity deficit model used. This instability rapidly generates vortical structures and induces turbulent motion in the central part of the wake.

As the vortex sheet rolls-up into the primary vortex, the part of the sheet forming the spiral is intensively stretched. Therefore, a longitudinal instability develops in the vortex sheet situated in the external periphery of the spiral. As it rolls-up, the instability grows rapidly. Away from the central part of the wake, below the primary vortex, the vortex sheet is also stretched and a similar longitudinal instability develops, probably triggered by the turbulent motion induced by the velocity deficit instability. Although similar, the wavelength and growth rate are different, respectively larger and lower. This is due to the larger thickness of the vortex sheet and to the weaker strain imposed by the primary vortex in this region, respectively. Subsequently, the growth of the former sinusoidal instability (in the periphery of the tip vortex) results in the generation of secondary vortices distributed longitudinally. These large scale structures are immediately wrapped around the primary vortices. As the roll-up goes on, the secondary vortices are intensively stretched and interact with the primary ones. Subsequently, the bursting of these vortical structures into small scales turbulence occurs.

Last but not least, a core instability develops. Its origin is likely a combination of the effects of the interaction with the surrounding vortical structures, and of the strong axial velocity component resulting from the roll-up of the velocity deficit and concentrated into the vortex cores. A characteristic helical deformation of the vortex core is then observed. This instability is likely to contribute to the well-known, yet open, meandering issue, observed in all wind tunnel investigations of trailing vortices. Indeed, similarly to the delta wing cases (mentioned in Section 1), the turbulence generated by the early instabilities developing in the vortex sheet shed at the wing trailing edge certainly contributes to the subsequent vortex meandering.
An interesting result concerns the fact that these three successive instabilities develop during the roll-up phase, within $\tau = 0.25$ (corresponding to a downstream distance of $x/b = 7.6$ in the present case). The flow field resulting from the roll-up phase thus consists in two vortices exhibiting helically deformed cores surrounded by weak turbulence. These highly complex three-dimensional mechanisms do not appear to modify significantly the longitudinally averaged vortex structure. Indeed, the vortex core size and vortex strength are very similar. The vortex trajectories and vortex structure (circulation and tangential velocity profiles), however, are somewhat affected by the surrounding turbulence and vortical structures, but not that significantly.

At higher Reynolds number, the same global dynamics occur. The weaker diffusion leads to the rapid development of instabilities, to a slower decay and a resulting enhanced effect of the turbulent secondary vortices on the primary ones.

The present work has thus provided extensive information concerning the temporal roll-up of a wing wake with velocity deficit. The next step would be to carry out the LES of the spatial-evolution of the same configuration. All the tools now exist: the efficient numerical method, the wake model and the corresponding temporal simulation for comparison. The main difficulty would be the large time and computational resources, as well as the manipulation of the large output files, that are required. Anyway, it would be a step further before the computation of the complete flow past a wing, foreseen in the near future with the VIC-PFM method.

Finally, the dynamics appeared to be comparable in some points to the one in jet/vortex interactions. The latter is also of great interest in aeronautical applications and is also investigated within the project FAR-Wake. The numerical simulation of the roll-up in presence of jets, using the same numerical tools, is ongoing work.
Figure 1: Initial axial vorticity field, $\omega_x t_0$, for the reference case (wake without velocity deficit).

Figure 2: Initial axial velocity profiles: (a) $(U - U_\infty)/U_\infty$ at $y/b = 0$ (the symbols represent the numerical field as discretized with the grid spacing $h$); (b) $(U - U_\infty)/U_\infty$ at $z/b = 0$. 
Figure 3: Initial velocity and vorticity fields for the velocity deficit configuration: 
(a) Axial velocity, $(U - U_\infty)/U_\infty$; (b) Spanwise vorticity, $\omega_y t_0$; (c) Vertical vorticity, $\omega_z t_0$; (d) transverse vorticity norm, $\omega_{yz} t_0 = (\omega_y^2 + \omega_z^2)^{1/2} t_0$. 
Figure 4: Longitudinally averaged axial vorticity field, $\omega_x t_0$, for the reference case (wake without velocity deficit): (a) $\tau = 0.02$; (b) $\tau = 0.05$; (c) $\tau = 0.1$; (d) $\tau = 0.25$. 
Figure 5: Longitudinally averaged axial vorticity field, $\omega_x t_0$, for the wake with added velocity deficit at $Re_\Gamma = 10^4$: (a) $\tau = 0.02$; (b) $\tau = 0.05$; (c) $\tau = 0.1$; (d) $\tau = 0.25$. 

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Figure 6: Longitudinally averaged transverse vorticity norm, $\bar{\omega}_{yz}t_0 = (\bar{\omega}_y^2 + \bar{\omega}_z^2)^{1/2}t_0$ (left), and axial velocity, $(U - U_\infty)/U_\infty$ (right), fields for the velocity deficit alone: (a) $\tau = 0.02$; (b) $\tau = 0.05$; (c) $\tau = 0.1$; (d) $\tau = 0.25$. 
Figure 7: Longitudinally averaged transverse vorticity norm, $\overline{\omega_{yz}} t_0 = (\overline{\omega_y^2} + \overline{\omega_z^2})^{1/2} t_0$, for the wake with added velocity deficit at $Re_{\Gamma} = 10^4$: (a) $\tau = 0.02$; (b) $\tau = 0.05$; (c) $\tau = 0.1$; (d) $\tau = 0.25$.  

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Figure 8: Isocontours of vorticity norm ($|\omega| t_0 = 400$ (high opacity) and $|\omega| t_0 = 200$ (low opacity)) colored by the axial component of vorticity, showing the time evolution of the three dimensional structure of the wake with velocity deficit at $Re_{\Gamma} = 10^4$. The wing flies towards the top-right corner of the figure; the $x$-axis thus points to the bottom-left corner.
Figure 8: (cont.)
Figure 8: (cont.)
$\tau = 0.1$

$\tau = 0.25$

Figure 8: (cont.)
Figure 8: (cont.)
Figure 9: Isocontours of vorticity norm ($|\omega| \ t_0 = 400$ (high opacity) and $|\omega| \ t_0 = 200$ (low opacity)) colored by the axial component of vorticity, showing the time evolution of the three dimensional structure of the wake with velocity deficit at $Re_\Gamma = 10^6$. The wing flies towards the top-right corner of the figure; the x-axis thus points to the bottom-left corner.
\[ \tau = 0.5 \]

**Figure 10:** (cont.)
Figure 11: Time evolution of (a) the longitudinally averaged vortex core positions and of (b) the longitudinally averaged effective vortex core radius: reference case at $Re_{\Gamma} = 10^4$ (dash), cases with velocity deficit at $Re_{\Gamma} = 10^4$ (solid) and at $Re_{\Gamma} = 10^6$ (dash-dot).

Figure 12: Time evolution of the longitudinally averaged normalized circulations, $\Gamma_{5-15}/\Gamma_0$: reference case at $Re_{\Gamma} = 10^4$ (dash), cases with velocity deficit at $Re_{\Gamma} = 10^4$ (solid) and at $Re_{\Gamma} = 10^6$ (dash-dot).
Figure 13: Longitudinally and azimuthally averaged profiles of circulation, $\Gamma(r)/\Gamma_0$ (left), and tangential velocity, $U_\theta(r)/V_0$ (right): reference case at $Re_\Gamma = 10^4$ (dash), and cases with velocity deficit at $Re_\Gamma = 10^4$ (solid) and at $Re_\Gamma = 10^6$ (dash-dot): (a) $\tau = 0.07$; (b) $\tau = 0.2$; (c) $\tau = 0.5$. 
Figure 14: Time evolution of the longitudinally averaged 2D energies: cross-flow kinetic energy, $E_c/E_{c0}$ (top), and axial-flow kinetic energy, $E_d/E_{c0}$ (bottom): reference case at $Re_\Gamma = 10^4$ (dash) and cases with velocity deficit at $Re_\Gamma = 10^4$ (solid) and at $Re_\Gamma = 10^6$ (dash-dot).

Figure 15: Time evolution of the linear impulses for the reference case at $Re_\Gamma = 10^4$: vertical impulse, $I_z/(\rho \Gamma_0 b)$ (dash) and axial impulse, $I_x/(\rho \Gamma_0 b)$ (dot), and for the case with velocity deficit at $Re_\Gamma = 10^4$: vertical impulse (solid) and axial impulse (dash-dot).
Figure 16: Longitudinally averaged axial velocity field, \((U - U_\infty)/U_\infty\), for the wake with added velocity deficit at \(Re_\Gamma = 10^4\): (a) \(\tau = 0.02\); (b) \(\tau = 0.05\); (c) \(\tau = 0.1\); (d) \(\tau = 0.25\).
Figure 17: Longitudinally averaged axial velocity field, \((U - U_\infty)/U_\infty\), for the wake with added velocity deficit at \(Re_\Gamma = 10^4\). \(\tau = 0.25\) like in figure 16d but with a re-scaled and saturated color scale. The white crosses indicate the vortex centers.
Figure 18: Longitudinally and azimuthally averaged axial velocity profiles, \( (U - U_\infty)/V_0 \): cases with velocity deficit at \( Re_\Gamma = 10^4 \) (solid) and at \( Re_\Gamma = 10^6 \) (dash-dot): (a) \( \tau = 0.07 \); (b) \( \tau = 0.2 \); (c) \( \tau = 0.5 \).
Figure 19: Fluctuations of axial vorticity, $\omega_x' t_0 = (\omega_x - \overline{\omega}_x) t_0$, taken at one longitudinal position at different times: (a) $\tau = 0.05$; (b) $\tau = 0.08$; (c) $\tau = 0.1$; (d) $\tau = 0.25$. The plots are centered on the vortex core center (marked by a black cross). The black circle corresponds to the vortex effective core radius.
Figure 20: Fluctuations of axial vorticity, $\omega'_z t_0 = (\omega_z - \omega_z') t_0$: same plot as in figure 19 but at $\tau = 0.2$, thus corresponding to the time between figure 19c and 19d.
References


