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3D filament simulations of Crow-like instabilities in ground effect

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3D filament simulations of Crow-like instabilities in ground effect

T. Lonfils, G. Daeninck and G. Winckelmans

April 26, 2006

1 Introduction

The purpose of this work is to investigate two vortex systems in ground effect (IGE) using a vortex filament method. The method being inviscid, the scope of this study is however reduced: 2-D and 3-D (DNS and LES) results clearly show that a very important effect for wake vortices (WV) IGE is the viscous interaction with the ground through the creation of secondary vortices. We therefore restrict the study to the analysis of long wave (Crow-type) instabilities for WV IGE.

After briefly describing the numerical method, we investigate two cases: (i) a configuration as close as possible to the one studied experimentally by CNRS-IRPHE (i.e., a thick vortex pair IGE with a forced long wave instability) and (ii) the case of aircraft like vortices IGE with a forced Crow instability (at different initial perturbation levels).

2 Vortex filament method

In 3-D inviscid flows, vortex lines move as material lines: this constitutes the basis for the method of vortex filaments. For more details on the vortex filament method, and vortex methods in general, refer to [3, 4]. Each filament $p$ corresponds to a vortex tube of circulation $\Gamma_p$: the strength associated to that filament. The method must be regularized, as singular filaments have a logarithmically infinite self-induced velocity everywhere their curvature is nonzero. For filaments using radially symmetric regularization functions, one has

$$u_\sigma(x) = -\frac{1}{4\pi} \sum_p \Gamma_p \int_{C_p} \frac{g_\sigma(|x - x_p|)}{|x - x|^3} (x - x_p) \times dx_p$$

where the regularization function $g_\sigma(r) \equiv g(\frac{r}{\sigma})$ is such that $g(\rho) \to 1$ for $\rho$ large and $g(\rho) \propto \rho^3$ for $\rho$ small. The corresponding streamfunction is obtained as

$$\psi_\sigma(x) = \frac{1}{4\pi} \sum_p \Gamma_p \int_{C_p} G_\sigma(|x - x_p|) dx_p$$

with $G_\sigma(r) \equiv \frac{1}{\sigma} G(\frac{r}{\sigma})$ and $\frac{d(\rho)}{\rho^3} = -\frac{1}{\rho} \frac{dG}{d\rho}(\rho)$. This streamfunction also corresponds to the solution of $\nabla^2 \psi_\sigma = -\omega_\sigma$ with, as vorticity field,

$$\omega_\sigma(x) = \sum_p \Gamma_p \int_{C_p} \zeta_\sigma(|x - x_p|) dx_p$$
where \( \zeta_\sigma(|x|) \overset{\Delta}{=} \frac{1}{4\pi\sigma} \zeta \left( \frac{x}{\sigma} \right) \). One has that \( g(\rho) = \int_0^\rho \zeta(s)s^2 ds \).

Typically, one uses parametric splines to represent numerically the filaments. The regularized method converges for regular vorticity fields when the number of filaments is increased, provided that the overlapping condition is satisfied. The regularization function has normalization \( \int \zeta_\sigma(x) \, dx = 1 \). It is of order \( r \) when it satisfies the moment properties \( \int x_1^{p_1} x_2^{p_2} x_3^{p_3} \zeta_\sigma(x) \, dx = 0 \) for \( 1 \leq p_1 + p_2 + p_3 < r \), and \( \int |x|^r |\zeta_\sigma(x)| \, dx < \infty \). For radially symmetric functions, these become \( \int_0^\infty \zeta(\rho) \rho^{2+s} \, d\rho = 0 \) for \( s \) even and \( 2 \leq s < r \), and \( \int_0^\infty |\zeta(\rho)| \rho^{2+r} \, d\rho < \infty \). Order higher than \( r = 2 \) calls for functions that are not strictly positive. See, Table 1 for usual functions. The low order algebraic function has \( r = 0 \): it can only be used for global vortex tube modelling (as done in the present work), as it does not converge for detailed field discretizations (using many filaments per vortex tube).

![Figure 1: Schematic of an infinite periodic filament and its image with respect to the vertical plane and to the horizontal plane (ground plane). The peak plane and the trough plane are also shown.](image)

The filament method is here also “filtered”: indeed, if nothing is done, a high wave number spurious mode will pollute the simulation (and eventually make it blow up). Here, we use a discrete filter acting on the splines representation of the filaments (as in [3]).

Figure 1 describes the geometry of the problem. The problem being periodic in the \( x \)-direction, we use a periodic version of the filament method. Planes of symmetry are forced following \( \hat{O}xy \), to take into account the inviscid wall, and following \( \hat{O}xz \) to ensure the symmetrical evolution of the vortices. We thus discretize and compute the evolution of one vortex filament only.

### 3 Definition

One defines the Fourier transforms of the filament position in the plane \( \hat{O}xy \) and \( \hat{O}xz \) respectively:

\[
\hat{y}_m = \frac{1}{N} \sum_{i=1}^{N} y(x_i)e^{-2\pi mi/N},
\]

\[
\hat{z}_m = \frac{1}{N} \sum_{i=1}^{N} z(x_i)e^{-2\pi mi/N},
\]
Table 1: Examples of 3-D \((g(\rho)\) and \(\zeta(\rho)\)) regularization functions. The circulation \(\Gamma(\rho)\) and vorticity \(\omega(\rho)\) distributions for an equivalent 2-D vortex (i.e. straight 3-D filament) are also provided.

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<th>(g(\rho))</th>
<th>(\zeta(\rho))</th>
<th>(\Gamma(\rho)/\Gamma_0)</th>
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<td>low order algebraic</td>
<td>(\frac{\rho^2}{(\rho^2+1)^{\frac{7}{2}}})</td>
<td>(\frac{3}{(\rho^2+1)^{\frac{5}{2}}})</td>
<td>(\frac{\rho^2}{(\rho^2+1)^{\frac{7}{2}}})</td>
<td>(\frac{1}{(\rho^2+1)^{\frac{7}{2}}})</td>
<td>0</td>
</tr>
<tr>
<td>high order algebraic</td>
<td>(\frac{\rho^2(\rho^2+5/2)}{(\rho^2+1)^{\frac{7}{2}}})</td>
<td>(\frac{15/2}{(\rho^2+1)^{\frac{7}{2}}})</td>
<td>(\frac{\rho^2(\rho^2+2)}{(\rho^2+1)^{\frac{7}{2}}})</td>
<td>(\frac{2}{(\rho^2+1)^{\frac{7}{2}}})</td>
<td>2</td>
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<tr>
<td>Gaussian</td>
<td>(\text{erf}(\rho) - \rho \frac{2}{\pi^{1/2}} e^{-\rho^2}) (\frac{4}{\pi^{1/2}} e^{-\rho^2}) ((1 - e^{-\rho^2}))</td>
<td>(\left(1 - e^{-\rho^2}\right))</td>
<td>(e^{-\rho^2})</td>
<td>(2)</td>
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where \(N\) the number of discrete points, \(m = 1, 2, \ldots, N/2\) and \(I^2 = -1\). The amplitude of each mode is:

\[ ||\hat{a}_m||^2 = ||\hat{y}_m||^2 + ||\hat{g}_m||^2 \]

The growth rate is then defined by:

\[ \sigma^* \triangleq \sigma t_0 = \frac{d}{dt^*} \log(||\hat{a}_m||) \]

where \(t^* = t/t_0\) and \(t_0\) is the characteristic time scale. This corresponds to the time for a pair of vortices characterized by an initial circulation \(\Gamma_0\) to descend OGE a distance \(b_0\) with the self-induced velocity \(V_0 = \frac{\Gamma_0}{2 \pi b_0}\).

4 Comparison: numerical simulation versus experiment

In the framework of FAR-wake Task 3.1.1, experiments have been performed by CNRS-IRPHE [1]. One of the aims of this work is to compare numerical results obtained using the filament method with experimental data. We first compare visualizations of vortex locations at different times. We then analyze the trajectories and temporal evolution of the vortex positions.

4.1 Experimental setup

The vortices are generated by two plates which are fixed to a common base and are able to rotate. The motion of the plates is properly chosen so as to have a spacing distance between the two vortices equals on average \(b_0 = 2.4 \text{[cm]}\). In order to force a longwave instability, the edges of the plates describe a sinusoid. Here, the wavelength and the amplitude of this sinusoid is respectively \(\lambda_f = 12 \text{[cm]} = 5.0 b_0\) and \(a_0 = 1 \text{[mm]} = 0.0417 b_0\). The initial moment is here defined when the vortex spacing no longer varies. This corresponds to \(h_0 = 6.2 b_0\). The experimental results show that the vortex circulation profile is relatively well fitted by a Gaussian vortex characterized by \(\sigma_g = 0.17 b_0\).

4.2 Numerical setup

In order to compare the numerical and experimental results, the numerical parameters must be carefully chosen in order to match the experimental configuration. The two vortices are separated by a distance \(b_0\) and are initialized at an altitude of \(h_0 = 6.2 b_0\). The perturbation
plane is put at 45°. In this plane, the perturbation is taken as:

\[ a(x) = a_0 \sin \left( \frac{2\pi x}{\lambda_f} \right) \]

where \( a_0 = 0.0417 b_0 \) (it turns out that this initial amplitude produces results that best match those of the experiment, see later) and \( \lambda_f = 5.0 b_0 \). Notice that this wavelength is significantly shorter than the natural Crow instability (with \( \lambda_C \approx 8 - 9 b_0 \)); it is nevertheless the one that was forced in the CNRS-IRPHE experiment and we here only aim at comparing with the experiment.

As mentioned above, the experimental vortex circulation profile can be fitted by a Gaussian vortex. However, the 3-D Gaussian regularization function is not implemented in the current filament code; this is because the Biot-Savart evaluation is expensive. To be able to compare two equivalent vortex evolutions, one uses vortices which have the same long wavelength dynamics.

In other words, in vortex filament methods, one can use vortices with another distribution function and require to obtain: the same translational velocity for thin vortex rings (Eq.3) and the same rotational velocity for long wavelength perturbations of a vortex tube (Eq.(4)). As shown below, those requirements are equivalent: the proper scaling fulfills both requirements.

• The self-induced translational velocity of a thin vortex ring \((\sigma/R \ll 1)\) is given, for a regularized vortex by

\[ U = \frac{\Gamma}{4\pi R} \left[ \log \left( \frac{8R}{\sigma} \right) - C \right] \tag{3} \]

with \( C = 1 \) for the low order algebraic function, \( C = 1/2 \) for the high order algebraic function and \( C = (1 - \gamma/2) = 0.7114 \) for the Gaussian function (where \( \gamma = 0.5772 \) is the Euler constant).

• The dispersion relation (self-induced rotational velocity) of long wavelength perturbations \((k\sigma \ll 1)\) is given, for a regularized vortex by

\[ \Omega = \pm \frac{\Gamma}{4\pi} k^2 \left[ \log \left( \frac{k\sigma}{2} \right) + \left( \frac{\gamma - 1/2}{2} \right) \right] + C \] \tag{4}

with the same \( C \) values as above.

We here use the high order algebraic distribution and we scale \( \sigma \) to obtain the same dynamics as those of a Gaussian vortex.

Another error comes from the fact that we here discretize a vortex tube with only one vortex filament: because the velocity is computed at the center of the filament, it is not equal to the true translational velocity of the vortex tube. It can be corrected by using, again, an equivalent \( \sigma \). Winckelmans [5] showed that using \( C = 0.5580 \) instead of 0.7114 in Eq.(3) and Eq.(4) leads to the correct dynamics of a Gaussian vortex characterized by \( \sigma_g \).

Thus, in order to obtain the same dynamics as those in the experiment, one simply uses the filament method with a high order algebraic regularization and with \( \sigma_{hoa} \) such that:

\[ \log \sigma_{hoa} + C_{hoa} = \log \sigma_g + C_g \]

with \( C_g = 0.5580 \), \( C_{hoa} = 0.5 \) and here with \( \sigma_g = 0.17 b_0 \). It leads to

\[ \sigma_{hoa} = 1.060\sigma_g = 0.180 b_0 \]

The vortex tube is represented by one vortex filament and is discretized by 320 points over \( \lambda_f \), and cubic splines. The time step is \( \Delta t^* = 10^{-2}/(2\pi) \). We use the same filter parameters as those detailed in [3].


4.3 Results

Comparison with the visualizations The qualitative vortex evolution is very similar to the experimental one. Figures 2 and 3 show the top and side views at $t^* = 2.7$ (during the linear growth phase) and $t^* = 3.8$ (at reconnection) respectively. Even if the Reynolds number of the experiment is not high ($Re \simeq 3500$ and thus there is some diffusion of the vortex core), the inviscid vortex filament method is in good agreement with the experimental results up to the reconnection phase: the position of the vortices and the amplitude of the instability are very similar.

![Figure 2: Comparison of the vortex wake evolution: numerical case (left) at $t^* = 2.7$ and experimental case (right) at $t^* \simeq 2.7$. (a) top view and (b) side view, at same scale. The tube radius shown in the simulation is that for which the vortex induced velocity is maximal: $r = r_c = 1.121 \sigma_g$.](image)

Temporal evolution up to reconnection Experimental results are given for characteristic sections: the peak and the trough sections: i.e., the sections where the altitude are maximal and minimal respectively (Figure 1). Up to the reconnection, trajectory, altitude and vortex spacing evolution are qualitatively similar as shown in Figure 4.(a) and Figure 5. The growth rate of the long wavelength instability is obtained as $\sigma^* = 0.66$, as shown in Figure 4.(b).
Figure 3: Comparison of the vortex evolution: numerical case (left) at $t^* = 3.8$ and experimental case (right) at $t^* \simeq 3.9$. (a) top view and (b) side view, at same scale. In the simulation, the tube shown is that for which $r = r_c$. 
Figure 4: Trajectory of the vortices (a) and evolution of the perturbation (b). The peak (solid line) and the trough (dash-dotted line).

Figure 5: Evolution of the spacing between the two vortices (a) and of the vortex altitudes (b). The peak (solid line), the trough (dash-dotted line) and the average (dotted line).

5 Analysis of the Crow instability in ground effect

Numerical simulations have been performed to investigate the behavior of the natural Crow instability in ground effect (before reconnection). Crow [2] has shown that the most unstable long wavelength mode of a pair of wake vortices ($\sigma/b_0 < 1$) has a wavelength: $\lambda = 8.6 b_0$. This mode grows in a plane inclined at about 48° to the horizontal. The growth rate is $\sigma^* = 0.83$. The purpose of this section is to investigate how the inviscid ground affects the Crow instability in terms of the deformation plane’s angle and of the growth rate.

Numerical setup The following simulations are performed using the low order algebraic regularization function (see Table 1) as it is a good fit usual for aircraft wake vortices. The
value of $\sigma$ is fixed to match a typical wake vortex core $\sigma = r_c = 0.05 b_0$. The vortices are initialized at an altitude out of ground effect: $h_0 = 6 b_0$. The perturbation is put in a plane inclined at 48.3° (accurate value determined by simulation carried OGE) to the horizontal and is initialized with a wavelength of $\lambda_C = 8.6 b_0$. Different perturbation levels were investigated: $a_0/b_0 = [10^{-1}, 10^{-2}, 10^{-3}, 10^{-4}, 10^{-5}, 10^{-6}]$. The filament is discretized using 512 points over $\lambda_C$, and cubic splines. The time step is $\Delta t^* = 5 \times 10^{-3}/(2\pi)$ and the filter parameters is again the same as those detailed in [3].

**Instability analysis**  Figure 6 and Figure 7 show the visualization of the filament evolutions for two initial perturbation levels: $a_0/b_0 = 10^{-2}$ and $a_0/b_0 = 10^{-3}$ respectively. One can see that there are essentially two cases: (i) when the initial perturbation is high enough, the vortex filaments reconnect before entering IGE, as in Figure 6 or (ii) for low initial perturbation level, the vortices enter IGE before reconnection and interact (and eventually reconnect) with their respective image.

Figures 8 and 9 describe the evolution of the amplitude and angle of the Crow mode for the different initial perturbation levels. Out of ground effect ($t^* < 4$), the growth rate of the Crow mode is $\sigma^* = 0.82$ and the perturbation angle is 48.3°. When the vortices are influenced by their respective images only ($t^* > 9$), the growth rate of the Crow mode is again $\sigma^* \simeq 0.82$; the perturbation angle is then $-41.7°$ (thus 48.3° with respect to the vertical). Between these two phases, the angle of the perturbation plane changes (Figure 9). Figure 10 further shows this phenomenon: $||\hat{z}_m||$ changes sign while $||\hat{y}_m||$ slows down, stops, and then grows again.

As expected, if the initial perturbation level is low enough, the results show that the evolution of the Crow instability IGE is universal and independent of the initial perturbation amplitude. The rotation of the perturbation plane also happens at the same altitude $z/b_0 \simeq 0.53$ above the ground (here at the same time $t^* \simeq 7.13$ as all simulations were initially at $h_0/b_0 = 6$). Figure 11 shows the trajectories of the vortices and the orientation of the perturbation planes. As expected, out of ground effect, and completely in ground effect, the instability characteristics (growth rate and angle) are those of the usual Crow instability.

**References**


Figure 6: Visualization of the vortex wake evolution. The shadow is also shown. The initial perturbation level is: $a_0/b_0 = 10^{-2}$.
Figure 7: Visualization of the vortex wake evolution. The shadow is also shown. The initial perturbation level is: \( a_0/b_0 = 10^{-3} \).
Figure 8: Temporal evolution of the amplitude for the Crow mode $\lambda_C/b_0 = 8.6$ for different initial perturbation levels.

Figure 9: Temporal evolution of the perturbation plane angle for each initial perturbation level $a_0/b_0$. The time at which the orientation changes sign is $t^* \simeq 7.13$, which corresponds to $z/b_0 = 0.53$. 
Figure 10: Temporal evolution of the amplitude for the Crow mode $\lambda C/b_0 = 8.6$. The initial perturbation level is: $a_0/b_0 = 10^{-6}$. $|\hat{a}|$ (solid line), $|\hat{y}|$ (dashed line) and $|\hat{z}|$ (dotted line).

Figure 11: Trajectories of the vortices and orientation of the perturbation planes (solid lines). Planes of symmetry (dash-dotted line). The time between each orientation shown is $\Delta t^* = 0.4$. 