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LES of two-vortex system in ground effect (longitudinally uniform wakes)

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This document reports about the large eddy simulation (LES) of a two-vortex system in ground effect, at a Reynolds of 20000. The present work, performed at CENAERO, was realized in close collaboration with the research group of Pr. Grégoire Winckelmans at UCL/TERM. Furthermore, the analysis is in the direct continuation of the direct numerical simulation (DNS) performed by Duponcheel et al. [3], at a Reynolds of 5000: the initial condition, the code and the post-processing tools are indeed the same. This similarity between the simulation parameters enables to clearly identify the influence of the Reynolds number.

Two “wall-resolved” LES, using different subgrid-scale (SGS) models, have been performed. The results will show the importance of the SGS modeling on the vortex system evolution. First, a LES with a Smagorinsky model supplemented by a Piomelli damping function in the near-wall
region is selected; the main reason being that this model is rather standard. Secondly, an advanced modeling strategy, based on a approach with flow scales discrimination [10], is investigated. This better method prevents the over-diffusive behavior of the classical Smagorinsky model in the vortex cores.

To present the aforementioned results, the remainder of this report is organized as follows. First, an introduction on the wake-vortex system in ground effect is provided. This section is extracted from the work of Duponcheel et al. [3]. Secondly, the numerical method is described, along with the modeling procedure. Third, the initial condition and the computational mesh are presented. Finally, the results are analyzed and differences with the $Re = 5000$ case are commented. The flow is described using 3D visualizations, as well as quantitative diagnostics (such as the energy evolution, the modal energy, the vortex trajectories and the $\Gamma_{5-15}$ circulation evolution).

1 Introduction

The first experimental observations of wake vortices in ground effect during flight test measurements took place in the late ’60. An upward motion of the vortices after their descent was observed. This phenomenon, called the rebound of the vortices, cannot be explained by the inviscid theory. With this assumption, a vortex dipole approaching a wall follows only an hyperbolic trajectory induced by the image vortices used to model the influence of the inviscid wall. In fact, the rebound is due to the separation of the boundary layer that develops on the no-slip wall. Indeed, this boundary layer is subjected to an adverse pressure gradient (deceleration) so that it finally separates and forms a secondary vortex. This opposite-sign vortex induces an upward velocity on the primary vortex and makes it rise.

As far as numerical simulations are concerned, Luton & Ragab [8] showed at a low Reynolds number ($Re = \Gamma_0/\nu = 2200$, where $\Gamma_0$ is the circulation of one primary vortex and $\nu$ is the kinematic viscosity) that the interaction of a straight vortex pair with a wall leads to a three-dimensional flow through a short-wavelength instability of the secondary vortex. They considered this instability as an example of the Widnall instability (named after Widnall et al. [15]). Moet [9] performed coarse large-eddy simulations (LES) at high Reynolds numbers ($Re \approx 3 \times 10^5$) and also observed the short-wavelength instability of the secondary vortex. Proctor & Han [13] performed very coarse LES at very high Reynolds numbers ($Re \approx 10^7$) corresponding to real aircraft cases. They showed that the decay of the vortices is much faster once the
vortices have reached their minimal altitude. The decay rate is only weakly influenced by the ambient turbulence level. Their results are in fair agreement with experimental observations at Dallas Airport. Proctor et al. [12] further investigated the sensitivity of the decay in ground effect to initial values of circulation, height, vortex separation and ambient turbulence level. The non-dimensional decay rate is found to be insensitive to these parameters and they propose a simple decay law.

In the study of Duponcheel et al. [3], a direct three-dimensional simulations (DNS) of a pair of vortices interacting with the ground was performed. As the simulations of Luton & Ragab [8] at a low Reynolds show only a weak interaction between the secondary and primary vortices, Duponcheel et al. decided to carry out their simulations at a higher Reynolds number, yet still low enough to avoid the need of a subgrid scale model, i.e., $Re = \Gamma_0/\nu = 5000$. Moreover, most of the previous researches investigated the interaction with the ground of vortices presenting core sizes that are larger than real aircraft vortices. Therefore, a realistic core size for the vortices was chosen.

The simulations showed that the secondary vortices are unstable, but the structure of the instability is somewhat different from the results of Luton & Ragab [8] due to the higher Reynolds number and the different core size. The elliptic instability causes a strong interaction of the secondary and primary vortices. This finally leads to a turbulent flow field and an enhanced decay rate compared to two-dimensional simulations. Nevertheless, the mechanism of the main instability was found to be similar to the one observed by Moet [9] in LES at higher Reynolds numbers (yet not “wall-resolved” LES). However, this fast decay phase occurs later than observed in the simulations of Moet [9], Proctor et al [12] or measured for real aircraft by Holzäpfel & Steen [4]. This can be partially explained by the lower growth rate of the instability due to lower Reynolds number.

In direct continuation of the DNS by Duponcheel et al. [3], the scope of the present work is to increase the Reynolds number. This is done using a “wall-resolved” LES. As the simulations have to be “wall-resolved”, the increased $Re$ is limited to 20000. In fact, the LES is carried out using the same code, a grid nearly equivalent to the DNS case and the same initial condition. This should enable to clearly investigate the influence of the Reynolds. As mentioned in the Duponcheel et al. [3] and Winckelmans et al. [16], a critical point is to choose a proper subgrid scale (SGS) model. The required SGS models are expected to be inactive during the gentle, well-resolved, phase of the flow, as well as in the laminar vortex cores, and to be active on high-wavenumbers, during the turbulent phase. To reach this objective, an advanced model, based on flow scales discrimination, was implemented and used (see Cocle et al. [1] and Winckelmans et al. [16]).
The LES at a $Re = 20000$ showed flow properties that present strong similarities with the DNS at $Re = 5000$. The short-wavelength instabilities also induce a strong interaction between the secondary and primary vortices, leading to a turbulent flow and an increased decay rate. Nevertheless, the decay phase occurs earlier than at $Re = 5000$. The increase of the Reynolds leads indeed to a **higher growth rate**. This Reynolds influence is thus in good agreement with the other aforementioned references [9], [12] and [4], where an earlier decay phase was reported. A comparison with a standard SGS model also shows the beneficial influence of the advanced modelling approach on the energy evolution. The $\Gamma_{5-15}$ evolution is also significantly different.

2 Numerical method

The governing equations are the Navier-Stokes equations for incompressible flows

\[
\begin{align*}
\frac{\partial u_i}{\partial x_i} &= 0, \\
\frac{\partial u_i}{\partial t} + \frac{\partial}{\partial x_j} (u_j u_i) &= - \frac{\partial P}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_j \partial x_j},
\end{align*}
\]

where $P = \frac{p}{\rho}$ is the reduced pressure, and $\nu$ is the constant kinematic viscosity. These equations are integrated in time using a fractional-step method with the pressure in *delta* form, as described by Lee et al. [6]. This form allows simple boundary conditions for the pressure and the intermediate velocity field. The convective term and the subgrid scale (SGS) model (resulting from the spatial discretization) are integrated explicitly using a second order Adams-Bashforth (AB2) scheme and the diffusion term implicitly using a second order Crank-Nicolson (CK2) scheme. The resulting linear system is decomposed using an ADI approximation, and solved with a LU direct solver. The second order time-stepping scheme reads

\[
\begin{align*}
\frac{u_i^* - u_i^n}{\Delta t} &= -\frac{1}{2} (3H_i^n - H_i^{n-1}) - \frac{\partial \phi^n}{\partial x_i} + \frac{1}{2} \frac{\partial^2}{\partial x_j \partial x_j} (u_i^* + u_i^n), \\
\frac{\partial^2 \varphi}{\partial x_i \partial x_i} &= \frac{1}{\Delta t} \frac{\partial u_i^*}{\partial x_i}, \\
\frac{u_i^{n+1} - u_i^*}{\Delta t} &= -\frac{\partial \varphi}{\partial x_i}, \\
\phi^{n+1} &= \phi^n + \varphi,
\end{align*}
\]
where $H^n_i$ stands for the terms integrated explicitly (convection and SGS model), $u^*_i$ is the intermediate velocity field and $\phi^n$ is the modified pressure.

The equations are discretized in space using the fourth order finite difference scheme of Vasilyev [14], in the skew-symmetric form. This discretization of the convective term conserves the discrete kinetic energy on Cartesian stretched meshes and is therefore particularly suitable for direct or large-eddy simulations of turbulent flows. The Poisson equation for the pressure is solved using an efficient multigrid solver with a Line Gauss-Seidel smoother. The code is designed to run efficiently in parallel.

The SGS stress tensor $\tau^M_{ij}$ is modeled using an eddy-viscosity model

$$\tau^M_{ij} = 2 \nu_t S_{ij} \quad \text{with} \quad S_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right).$$

The eddy-viscosity, $\nu_t$, is computed with the Smagorinsky model. Nevertheless, this model does not produce a $\nu_t$ with the good near-wall behavior, $(y^+)^3$. To reach this objective, the Smagorinsky constant, $C_s$, is scaled by a damping function, $F_p$, following Piomelli [11],

$$\nu_t = (C_s F_p) \Delta^2 (2 S_{ij} S_{ij})^{1/2},$$

$$C_s = 0.027, \quad F_p = 1 - \exp(-((y^+)^3/25)),$$

where $\Delta = (h_x h_y h_z)^{1/3}$ with $h_i$ the grid size in the $i$ direction. Another solution would be to implement the WALE model of Nicoud et al. [10], as it presents naturally the good near-wall behavior. However, the WALE model is known to be more dissipative than the Smagorinsky in vortex cores. Although less dissipative, the Smagorinsky still presents a significant (and to high) dissipation in vortex cores. To avoid this excessive dissipation, a second eddy-viscosity model has been implemented. This advanced model is based on a flow scales discrimination, where the SGS viscosity is evaluated using a fluctuating velocity field, $u'_i$. These fluctuations, termed small field, are computed from the difference between the complete velocity field, $u_i$, and a filtered velocity field, $\bar{u}_i$. In the present work, the filtered field is obtained by the multiple application of three-points discrete filters [1], [16]. For the sake of completeness, the advanced model is here called small-complete model, as the eddy-viscosity is computed on $u'_i$ and the strain rate tensor on the complete velocity, $u_i$. The damping function is still necessary. Let’s mention that the performance of this SGS methodology combined with the present discretization approach has also been validated successfully on the turbulent channel flow at $Re_\tau = 395$. 
3 Numerical setup

The flow consists in a pair of counter rotating vortices evolving under their mutual influence and interacting with the ground in a viscous way. This simulates quite realistically the wake vortex system in the far field of an aircraft flying close to the ground (after completion of the initial roll-up). A careful choice of the computational and physical parameters for such a flow is mandatory and was investigated in [2].

3.1 Computational domain

Defining the initial distance between the vortex centers as $b_0$, the lengths of the domain are $L_x = 4 b_0$, $L_y = 8 b_0$ and $L_z = 3 b_0$. The initial height of the vortex pair is set to $h_0 = b_0$, as illustrated in Fig. 1. The number of grid points are $N_x = 256$, $N_y = 512$ and $N_z = 256$, leading to a total of 33 millions of nodes. The grid is quite different from that of DNS. The grid is indeed more refined in the wall-normal direction to enable the first off-wall grid point to be located at a $y^* \sim 1$ distance from the wall. Furthermore, the growing factor remains compatible with a wall-resolved LES. Given the number of nodes, the simulation was performed in parallel, on 16 processors (using the CENAERO cluster, “tsunami”, equipped with Intel Xeon processors at 3 GHz, each with 1 Gb of RAM).

The domain is periodic in the $x$ and $y$ directions. A no-slip condition is set at the ground ($z = 0$) and a slip condition at the top of the computational domain ($z = 3 b_0$).

![Figure 1: Sketch of the computational domain. The domain is extruded in the x direction. The circles represent the location of the vortices.](image-url)
3.2 Initial condition

The simulation of a wake vortex system requires to choose a distribution function to initialize the vortices. As in the DNS of Duponcheel et al. [3], the low order algebraic profile was used. This profile is defined as

\[
\omega(r) = \frac{\Gamma_0}{\pi} \frac{r^2}{(r^2 + r_c^2)^2}, \\
\Gamma(r) = \frac{\Gamma_0 r^2}{(r^2 + r_c^2)}, \\
u_\theta(r) = \frac{\Gamma(r)}{2\pi r},
\]

(10) (11) (12)

where \(\Gamma_0\) is the circulation of the vortex and \(r_c\) is the radius of maximum induced tangential velocity. It is set here to \(r_c = 0.05 b_0\). The Reynolds number of this flow is set to \(Re = \Gamma_0/\nu = 20000\). The velocity scale for this problem is based on the initial descent velocity of the vortex pair out of ground effect,

\[V_0 = \frac{\Gamma_0}{2\pi b_0}.
\]

(13)

This also defines the non-dimensional time \(t^* = t \frac{V_0}{b_0}\). The initial velocity, \(u^{BS}\), at each point of the \(Oyz\) plane is computed according to the Biot-Savart law, taking into account a finite number (typically 50) of image vortex pairs, due to the periodicity in the spanwise direction \((y)\) and the mirror vortex pairs in the direction perpendicular to the ground \((z)\). The 2D field is then extruded in the \(x\) direction. Finally, a 3D random perturbation is added to this velocity field:

\[u = u^{BS} + u_\theta(r_c) \lambda.
\]

(14)

The amplitude of the perturbation \(\lambda\) is a random variable with uniform distribution in \([-0.001, 0.001]\). This perturbation level is 10 times lower than the value reported in the paper of Duponcheel et al [3]. However, the further DNS results of Duponcheel mentioned in the present report have been computed using an equivalent perturbation level. The present DNS and LES initial conditions are therefore equivalent. The initial condition is pictured on Fig. 2 along with the simulation domain (partitioned in 16 subdomains), Fig. 3 represents the initial vortices (represented by circles of radius \(r_c\)) embedded in the mesh.
Figure 2: Sketch of the computational domain and of the partition into 16 subdomains. The visualization of the initial flow field is performed using isosurfaces of $\|\omega\|b_0^2/\Gamma_0 = 1$ and 10.

Figure 3: Two-dimensional view of a part of the computational mesh, and initial position of vortices (circles of radius $r_c$)

4 Results

4.1 Description of the flow

In this section, the flow is only described for the better small-complete model using three methodologies: first, by way of vorticity isosurfaces (Fig. 4), secondly, by the $\lambda_2$ criterion (Fig. 7), and third, by the mean axial vorticity (Fig. 8).
Figure 4: Visualization of the flow field using isosurfaces of $\|\mathbf{\omega}\|_b^2/\Gamma_0 = 1$ and 10 (small-complete model).
Figure 4: Visualization of the flow field using isosurfaces of $\|\omega\|b_0^2/\Gamma_0 = 1$ and 10 (small-complete model).
Figure 4: Visualization of the flow field using isosurfaces of $\|\omega\|b_0^2/\Gamma_0 = 1$ and 10 (small-complete model).
Figure 5: Close-up of the flow field using isosurfaces of \( \| \omega \| b_0^2/\Gamma_0 = 1 \) and 10 (small-complete model).
Figure 6: Top view visualization of the instability structure at $t^* = 2.19$, using isosurfaces of $\|\omega\|b_0^2/\Gamma_0 = 1$ and 3.5
Figure 7: Visualization of the flow field using the $\lambda_2$ criterion (small-complete model).
Figure 7: Visualization of the flow field using the $\lambda_2$ criterion (small-complete model).
Figure 7: Visualization of the flow field using the $\lambda_2$ criterion (small-complete model).
Figure 8: Longitudinally averaged axial vorticity field $\omega_x$ (small-complete model).
Figure 8: Longitudinally averaged axial vorticity field $\omega_x$ (small-complete model).
During the first stage, the vortex pair descends due to the mutually induced velocity. The impermeability condition at the wall can be modeled by a mirror vortex pair, resulting in a hyperbolic trajectory. However, as the induced tangential velocity near the wall is nonzero, the no-slip condition is at the origin of the development of boundary layers beneath the vortices. Those are clearly visible in Figs. 4(a), 7(a), 8(a). Starting from the stagnation point on the symmetry plane between the vortices, the pressure gradient forcing the boundary layers is first favorable and then adverse (acceleration followed by deceleration). This leads to their separation, when the primary vortices are close enough to the wall. The detached vortex-sheet then rolls-up and forms two coherent secondary vortices (see Figs. 4(b), 7(b), 8(b)). They induce an upward velocity on the primary vortices and force the latter to rise; this is the rebound phenomenon.

At this time, the flow is still essentially two-dimensional. But the secondary vortices are unstable with respect to short-wavelength modes. These perturbations are growing while the secondary vortex is orbiting around the primary vortex, as can be seen in Figs. 4(c), 7(c), 8(e). The dipole formed by the primary and secondary vortices is shown in Fig. 6, where an different isosurface has been drawn revealing the deformation of the core of the secondary vortex. As in the DNS at $Re = 5000$, it is characterized by the opposite deformation of the vortex inner core and of the outer layers, with one neutral surface in between.

The evolution of the instability (Figs. 4(d), 7(d), 8(d)) is explained by Leweke & Williamson [7] as a consequence of the deformation structure of the secondary vortex. Indeed, at some places, the outer layers of the secondary vortex come close enough to the primary vortex to be captured by the latter. A close-up of this phenomenon is pictured on Fig. 5(a). As they orbit around it, they are stretched and their intensity increases, leading to elongated tube-shaped structures. These structures are significantly less coherent than the loops described in the $Re = 5000$ benchmark. They tend to bend the primary vortex but they don’t reconnect with it. They are further stretched but new vortical structures are created from the secondary vortex (Figs. 4(e), 7(e), 8(e)). The flow field is now very complex and fully three-dimensional (Figs. 4(f), 7(f), 8(f)). The secondary vortex has lost its coherence and its vorticity is distributed around the primary vortex, which is also very disturbed. The flow finally evolves to a fully turbulent vortex system, Figs. 5(b), 8(h).

During the earlier stages of the simulation, a new set of weaker secondary vortices have been shed (Figs. 4(d), 7(d), 8(d)). These two co-rotating vortices merge, the second vortex rotating around the first (Figs. 8(f), 8(g)). At the end of the simulation, the resulting vortex is destroyed by the global turbulent
flow generated by the major secondary and primary vortices (Figs. 5(b) and 8(h)). The turbulence is also located near the ground. As a consequence, no further coherent structures are emitted from the wall. This can also be explained by the location of the primary vortices which are distant from the wall.

4.2 Analysis of the vortex trajectory

Figure 9: Trajectories of the vortices: small-complete model (solid) and Smagorinsky model (dash-dot) at $Re = 20000$, DNS at $Re = 5000$ (dot), with a line marker any $1.0t^*$. 

Fig. 9 compares the trajectories for the center of the right-hand side primary vortex, as obtained by the DNS and two LES simulations. The center is defined as the maximum of the longitudinally averaged axial vorticity. During the first part of the descent, all the trajectories are equivalent, as they correspond to the inviscid theory. As soon as the boundary layer starts to separate, the rebound occurs and the trajectories are significantly different between the DNS at $Re = 5000$ and the two LES at $Re = 20000$. The loop trajectory induced by the rebound presents a smaller radius at $Re = 20000$.

From Duponcheel et al. [3], the $Re = 5000$ trajectory is hitherto equivalent to results from two-dimensional simulations. Nevertheless, after $t^* \simeq 3$, corresponding to the strong 3-D interaction between the secondary and primary vortices, the velocity induced by the secondary vorticity is totally modified in the 3-D case, because the secondary vortex has lost its coherence. The
computed trajectory is irregular as the primary vortex is turbulent so that the position of the maximum of vorticity fluctuates around a mean position. Moreover, the initial perturbation being asymmetric, asymmetric trajectories are also observed. At $Re = 20000$, the trajectory is less irregular and is nearly independent of the SGS model used.

Numerical two-dimensional and three-dimensional investigations [2] have also shown that the trajectories are far from being independent of the domain size, because the boundary conditions affect significantly the flow and especially the boundary layers. They allowed to show that the confinement effect for the domain we use here is sufficiently low and that it affects only very weakly the instability mechanism.

### 4.3 Analysis of the vortex strength

The analysis of the vortex intensity is an important issue in scope of the *wake vortex problem*. Different criteria exist. The first one is the kinetic energy of the system, defined as

$$E = \frac{1}{2} \int_{\Omega} \left( u_x^2 + u_y^2 + u_z^2 \right) d\Omega, \hspace{1cm} (15)$$

where $\Omega$ is the domain. For sufficiently large computational domain, as used here, it does not depend on the domain size. Another usual measurement of the vortex intensity is $\Gamma_{5-15}$, which is an average of the circulation distribution. It was originally defined as

$$\Gamma_{5-15} = \frac{1}{10} \int_5^{15} \Gamma(r) \, dr, \hspace{1cm} (16)$$

for aircraft with a wingspan $b = 60 \ [m]$, where $\Gamma(r)$ is the circulation distribution in a disk of radius $r$ centered on the vortex. In fact, $\Gamma_{5-15}$ gives a good information about the strength of the rolling moment induced by a wake vortex on another aircraft. One defines the $\Gamma_{5-15}$ for any type of aircraft by

$$\Gamma_{5-15} = \frac{1}{b} \int_{b_0}^{b} \Gamma(r) \, dr \frac{1}{b_0} \int_{b_0}^{b_0} \Gamma(r) \, dr, \hspace{1cm} (17)$$

where $s$ is the ratio $b_0/b$: a typical value being $\pi/4$ for an elliptic loading. The evaluation of $\Gamma(r)$ is here obtained by integrating the mean axial vorticity field $\omega_x$ in a disk of radius $r$

$$\Gamma(r) = \int_0^{2\pi} \int_0^r \omega_x(r') \, r' \, dr' \, d\theta'. \hspace{1cm} (18)$$
Fig. 10 describes the evolution of the kinetic energy. The small-complete model being inactive during the first laminar part of the simulation, the
curves of the 2-D and small-complete model LES are very close. On the contrary, the classical Smagorinsky model removes a relevant part of the resolved kinetic energy. This is the major motivation that lead us to the implementation and use of an advanced SGS model, and to only report flow pictures for this case.

From $t^* \simeq 2.8$, the energy evolution departs from the 2-D case. In fact, the dissipation rate of the 3-D case increases dramatically because the interaction between the secondary and primary vortices generates small tridimensional structures. Although the model choice plays an important role on the energy level, there is no significant influence on the fast decay starting time. Furthermore, when high wavenumbers become saturated, the dissipation rate is found to be equivalent between both models.

Compared to the $Re = 5000$ evolution, the fast decay occurs earlier, $t^* \simeq 2.8$ instead of $t^* \simeq 3.5$ and the transition between both regimes is sharper. As a consequence, the growth rate of the tridimensional instabilities should be more important.

The evolution of the $\Gamma_{5-15}$ (Fig. 11) is quite different. The initial 2-D decay phase is governed by the molecular viscosity and is due to the slow spreading of the core size. The increase of the Reynolds number from 5000 to 20000 is beneficial as it leaves this first decay phase negligible (it was not the case for $Re = 5000$). Let’s mention that the curves of the two SGS model are seen to be quite close.

As for the energy evolution, the fast decay phase starts at $t^* \simeq 2.8$, and independently of the SGS model. This starting time happens significantly earlier than in the DNS at $Re = 5000$. This was also observed by Moet [9], Proctor et al [12] and also by Holzäpfel & Steen [4]. The fast decay rate of the classical Smagorinsky model is at first more important than the small-complete model, and finishes at the same level. The $\Gamma_{5-15}$ being an important hazard criterion, the previous behavior again emphasizes the importance of proper subgrid scale modeling.

### 4.4 Modal Analysis

The modal analysis is particularly important in order to understand the actual difference between the two-dimensional evolution and the three-dimensional one. Three-dimensional instabilities described above come from the deformation of the primary and the secondary vortices. These instabilities grow and evolve in time. This can be properly investigated by a modal analysis.

In the modal space, the discrete Fourier transform of the velocity field in
the axial direction is

$$\tilde{u}(k_m, y_j, z_l) = \frac{1}{N_x} \sum_{i=1}^{N_x} u(x_i, y_j, z_l) e^{-i k_m L_x / N_x},$$

(19)

where $I^2 = -1$ and $k_m$ is the wavenumber (with $m = 1, 2, \ldots, N_x/2$). The discrete kinetic energy is thus, in the Fourier domain,

$$\tilde{E}(k_m, y_j, z_l) = \frac{1}{2} \left( |\tilde{u}_x(k_m, y_j, z_l)|^2 + |\tilde{u}_y(k_m, y_j, z_l)|^2 + |\tilde{u}_z(k_m, y_j, z_l)|^2 \right).$$

(20)

The average energy of each mode is the axial direction is then

$$E(k_m) = \frac{1}{L_y L_z} \sum_{j=1}^{N_y} \sum_{l=1}^{N_z} \left( \tilde{E}(k_m, y_j, z_l) \Delta y_j \Delta z_l \right).$$

(21)

The wavelength of each mode is

$$\lambda_m = \frac{L_x}{k_m} = \frac{2\pi}{k_m}.$$ As in Laporte [5], one also defines the growth rate $\sigma$ of the instabilities by:

$$\sigma^* = \sigma t_0 = \frac{1}{2} \frac{d}{dt^*} \log \left( \frac{E(k_m)}{E_0(0)} \right),$$

(22)

where $E_0(0)$ is the modal energy of the base mode at time $t^* = 0$.

Fig. 13 describes the evolution of the modal energy for some characteristic wavenumbers: i.e, $k b_0/(2\pi) = b_0/\lambda = [0; 0.5; 1.5; 2.75; 4]$. During the descent, all the modes are stable and their energy decreases. When the separation of the boundary layer occurs ($t^* \approx 1.0$), some wavenumbers become unstable. In the DNS of Duponcheel et al. [3], the growth rate $\sigma^*$ of the instabilities is approximatively equal to $\sim 3$. In the LES at $Re = 20000$, the Fig. 13 shows a growth rate of $\sim 4.5$.

Fig. 12 shows the spectrum of the modal energy at different times. Before $t^* \approx 3.0$, a peak is clearly visible at $k b_0/(2\pi) = 3.5$ for the small-complete model. It corresponds to the short-wavelength instabilities depicted in Fig. 6. This peak is not as clear for the Smagorinsky model. Compared to the $Re = 5000$ test case, the spectra are also more spread (wider bands) for each SGS model. Indeed, the interaction between the secondary and primary vortices leads quickly to very complex tube-shaped structures which are less coherent than the loops found at $Re = 5000$. All the modes saturate at the time $t^* \approx 3.0$ when the instabilities are well developed. At the end of the simulation, the spectrum is smooth. This corresponds to a fully turbulent flow.
Figure 12: Modal energy spectra $E(k)/E_0(k=0)$ at $t^* = 0.86 (\times), 1.53 (\circ), 2.48 (~), 3.25 (\Box)$ and $5.63 (*)$.

5 Conclusion

In this study, the large eddy simulation (LES) at a Reynolds of 20000 of a two-vortex system in ground effect have been performed. This is in the direct continuation of the direct numerical simulations (DNS) of Duponcheel et al. [3]. The simulation parameters being almost equivalent (the same ini-
Figure 13: Time history of the modal energy $E(k)/E_0(k = 0)$ for the modes $kb_0/(2\pi) = b_0/\lambda = 0$ (*), 0.5 (○), 1.5 (□), 2.75 (◇) and 4 (△).

Initial condition, the same code and almost the same mesh), the present work intends to investigate properly the influence of the Reynolds number.

The results showed that the modeling strategy plays an important role. The impact was indeed significant on the kinetic energy and $\Gamma_5−15$ evolutions. In view of the diagnostics based on energy, circulation and modal energy, the small-complete model, based on a flow scales discrimination, clearly out-
performs the Smagorinsky model. Indeed, as expected, the small-complete model is inactive during the gentle, laminar phase of the simulation. For this reason, the figures describing the flow structure are only reported for the better results: those obtained with the small-complete model.

The $Re = 5000$ and $20000$ flows present strong similarities, also in term of instability mechanisms, even though the secondary and primary vortices interaction at $Re = 20000$ generates rapidly very complex tridimensional vortical structures. As expected, the increase of the Reynolds number leads to a fast decay phase that happens earlier, and that presents a higher growth rate.

It is also expected that the present LES results (energy and circulation decay rate, vortex trajectories), are already at sufficiently high Reynolds number that they are representative of what should be expected at much higher values, relevant to aircraft wakes. This could be further confirmed by comparisons to qualify lidar data measured on ground effects.

References


