AST4-CT-2005-012238

**FAR-Wake**

Fundamental Research on Aircraft Wake Phenomena

Specific Targeted Research Project

Start: 01 February 2005
Duration: 40 months

**Water tank experiments on vortex pairs IGE**

Prepared by: C. Cottin (CNRS-IRPHE, UCL)
T. Leweke (CNRS-IRPHE)

<table>
<thead>
<tr>
<th>Deliverable No.:</th>
<th>TR 3.1.1 - 4</th>
<th>Due date:</th>
<th>January 2008 (m36)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Version:</td>
<td>1.1</td>
<td>Task manager:</td>
<td>G. Winckelmans</td>
</tr>
<tr>
<td>Date delivered:</td>
<td>25 April 2007</td>
<td>Project manager:</td>
<td>T. Leweke</td>
</tr>
<tr>
<td></td>
<td></td>
<td>EC Officer:</td>
<td>S. Stoltz-Douchet</td>
</tr>
</tbody>
</table>

Dissemination Level

<table>
<thead>
<tr>
<th>PU</th>
<th>Public</th>
</tr>
</thead>
<tbody>
<tr>
<td>PP</td>
<td>Restricted to other programme participants (including the Commission Services)</td>
</tr>
<tr>
<td>RE</td>
<td>Restricted to a group specified by the consortium (including the Commission Services)</td>
</tr>
<tr>
<td>CO</td>
<td>Confidential, only for members of the consortium (including the Commission Services)</td>
</tr>
</tbody>
</table>

Project co-funded by the European Commission within the Sixth Framework Programme (2002-2006)
ABSTRACT

The dynamics of longitudinally uniform counter-rotating vortex pairs in the vicinity of the ground have been investigated experimentally. The vortices were generated in a water tank by the impulsive rotation of long flat plates, and the flows were characterised by visualisations using Laser-Induced Fluorescence, as well as by quantitative measurements by Particle Image Velocimetry. With respect to the application to aircraft wakes near the ground, the following aspects are of particular interest and were addressed in this study: vortex trajectories, evolution of the circulation, and the mechanisms leading to vortex decay. Investigations were carried out for two initial heights of the vortex pairs above the ground. The main part of this work concerns the interactions of a straight vortex pair generated at a height of twice the vortex separation distance, for three different values of the circulation-based Reynolds number ($Re_Γ = 1900, 3500$ and $5200$). The results show how the decay of the primary vortices, through interaction with the secondary ones forming from the opposite-signed vorticity lifting off the ground, is greatly enhanced when increasing $Re_Γ$. Details on the development of the short-wavelength instability of the secondary vortex and its effect on the overall dynamics are given. The development of a new centrifugal-type instability, observed at higher Reynolds number and attributed to the vortex generation system, is discussed. Investigations at a high vortex generation height (six vortex spacings) have also been carried out for two Reynolds numbers ($Re_Γ = 3000$ and $4000$) in order to validate the flow conditions (vortex generation, uniformity, Crow instability development). The interactions of periodic vortex rings resulting from the late stages of Crow instability show interesting new features in comparison to the case of a single vortex ring impacting a wall.

Key words:

Counter-rotating vortex pairs, ground interactions, rebound, three-dimensional instability.
CONTENTS

Abstract ...................................................................................................................... 2

Contents ......................................................................................................................... 3

List of figures ............................................................................................................... 4

1 Introduction .......................................................................................................... 6

2 Vortex pair dynamics ........................................................................................... 7

   2.1 Basic considerations ....................................................................................... 7
   2.2 Dynamics “out of ground effect” (OGE) ........................................................ 8
   2.3 Dynamics “in ground effect” (IGE) .............................................................. 11
      2.3.1 Two-dimensional interactions ............................................................. 11
      2.3.2 Three-dimensional interactions ........................................................ 12
   2.4 Objectives of the present work ................................................................. 15

3 Experimental details............................................................................................ 16

   3.1 Experimental set-up ....................................................................................... 16
   3.2 Laser-Induced Fluorescence technique ........................................................ 20
   3.3 Particle Image Velocimetry .......................................................................... 22
   3.4 Initial flow field ............................................................................................ 23

4 Results .................................................................................................................. 26

   4.1 Two-dimensional dynamics........................................................................... 26
   4.2 Three-dimensional dynamics........................................................................ 36
      4.2.1 Centrifugal-type instability ..................................................................... 41
      4.2.2 Second short-wavelength instability .................................................... 44
   4.3 Crow instability development for large initial heights and interaction with the ground ................................................................. 49
   4.4 Summary and discussion .............................................................................. 56

5 Conclusion ............................................................................................................ 58

References .................................................................................................................... 59
LIST OF FIGURES

Figure 1: Crow instability observed in the wake of a transport aircraft showing the 45° plane of the long wavelength instability. ................................................................. 10
Figure 2: Vortices in the wake of civil transport aircraft. ..................................................................................................................................................................................... 10
Figure 3: Long- and short-wavelength instabilities in a vortex pair Re=2750 (Leweke & Williamson, 1998a). ........................................................................................................................................ 10
Figure 4: Experimental apparatus. ................................................................................................................................................................................................. 16
Figure 5: Bottom of the water tank showing the devices used to avoid end effects and to position the vertical ground. ........................................................................................................ 17
Figure 6: Top view of the vortex generator .................................................................................................................................................................................. 18
Figure 7: Sketch of the cross-section of the vortex generator showing the plates (B = 10 cm), the common base (S = 5.1 cm) and the definition of the position angle. ...................................................... 19
Figure 8: Qualitative shape of the plate motion history: (a) angular velocity [°/s] of the plate as function of time [s]; (b) mean flow velocity [cm/s] between the plate tips ........................................ 20
Figure 9: Cross-section visualizations of the vortex pair: (a) the peak and trough planes are visualized simultaneously using two parallel laser sheets; (b) case of a pair of straight vortices visualized with a single light sheet. The plate tips are visible above the vortex pairs. .......................................................... 21
Figure 10: Visualization of Crow instability in cross-sections. (a) Visualization set-up with peak plane laser sheet (1) and trough plane laser sheet (2); (b) Schematic of the simultaneous view of the two planes (see figure 9a). ........................................................................................................................................ 22
Figure 11: Initial (t* = 0) vorticity field [s⁻¹] obtained from PIV measurements for Re=5200. .................................................................................................................. 23
Figure 12: Radial profiles of azimuthally averaged swirl velocity and circulation (Γ = 2πrV₀), for Re=5200. Symbols represent measurements from PIV, solid lines are least-squares fits to the theoretical profiles of a Lamb-Oseen vortex (fit parameters: Γ = 50.8 cm²/s, a = 0.433 cm). The dotted red line represents r = b₀/2. .......................................................................................................................... 24
Figure 13: Dependence of the initial vortex spacing on the initial Reynolds number. .......................................................................................................................... 25
Figure 14: Evolution of vortex parameters for Re=5200. (a) dimensional units, (b) non-dimensionalised with b₀, the theoretical relation rᵣ = 1.12a for a Gaussian vortex has been used. h: mean distance between the vortices and the wall; rᵥ: mean core radius; a: mean core radius for a Gaussian vortex; Γ: mean circulation. The black dotted line corresponds to the theoretical core growth of a Lamb-Oseen vortex. .................................................................................................................................................. 25
Figure 15: Vorticity fields for Re=5200 and b₀ = 2b₀ of one vortex with corresponding cross-cut visualisations at successive non-dimensional times. .......................................................................................................................................................... 27
Figure 16: Same as figure 15 at later times. .................................................................................................................................................................................. 28
Figure 17: Trajectories for three different Reynolds numbers. Figures on the right side show non-dimensional and symmetrised trajectories where x/b₀ = 0 corresponds to the symmetry axis and the y/b₀ values are obtained by averaging the y-position of the two primary vortices. .................................................................................................................................. 29
Figure 18: Maximum non-dimensional time before bursting of the primary vortex, as function of Reynolds number. .............................................................................................................................. 30
Figure 19: Primary vortex centre trajectories for different Reynolds numbers. (a) Re=1900, t*ₙₜₜ = 11.8; (b) Re=3500, t*ₙₜₜ = 7.5; (c) Re=5200, t*ₙₜₜ = 5.0. .................................................................................................................. 31
Figure 20: Vertical position of the primary vortices. t* = 0 is defined as the time at which h₀ = 2b₀. ........................................................................................................................................................... 32
Figure 21: Evolution in time of the primary vortex circulation (diamonds), of the total mid-plane circulation (circles), and of the difference between the two formers (squares). Only one half of the total domain is used for the computation (including one primary vortex only). This half-plane corresponds to the domain of the vorticity fields shown in figures 15 and 16. Circulations are non-dimensionalised by Γ₀. .................................................................................................................................. 33
Figure 22: Average (over the three Reynolds numbers) circulations of the primary vortex and opposite-signed vorticity at different non-dimensional times. $\Gamma_0$ is the initial ($h_0 = 2b_0$) circulation of the primary vortex.

Figure 23: (a) Vorticity field for $Re_T = 5200$ at $t^* = 3.6$ showing the line joining the vortex centres. (b) Azimuthal (normal to the line) velocity [cm/s] as function of the position [cm] along this line. The origin is the middle point between the two vortex centres.

Figure 24: Iso-contours of vorticity obtained by DNS (Duponcheel et al. 2006) at $t^* = 3.96$ for $Re_T = 5000$. The initial height is $b_0$, the longitudinal extent of the computational domain is $4b_0$ and the initial perturbations are white noise (random and very small amplitude).

Figure 25: Same configuration as shown on figure 24, except with a longitudinal extent of $2b_0$.

Figure 26: Side view visualizations, only the farther vortex is visualized. The first row corresponds to the initial time $t_0$ ($h_0 = 2b_0$), the second row to the minimum distance from the ground ($t^* = 2$), the third row to the maximum rebound ($t^* = 4.5$), and the last row ($t^* = 6.4$) shows a later time. The top of each image corresponds to the plate level, the bottom to the ground.

Figure 27: Longitudinal cross-section of one of the two primary vortices, showing the short wavelength centrifugal-type instability developing at its periphery. The laser sheet used for the LIF visualization is parallel to the ground and positioned at about $h = 0.6b_0$ (height of primary vortex minimum rebound, i.e. $t^* = 2$). The secondary vortex is not visualized. $Re_T = 5200$.

Figure 28: Simplified sketch of a longitudinal section of the primary vortex at late times showing cross-sections of the secondary vortex and the resulting deformation of the primary one.

Figure 29: Flowfield for $Re_T = 5200$ and $h_0 = 2b_0$, at $t^* = 1.2$. (a) Vorticity field. (b) Profiles of azimuthally averaged circumferential velocity and circulation ($F_r = 2\pi r V_\theta$) of the primary vortex.

Figure 30: (a) local inviscid growth rate $\sigma = (-\Phi)^{1/2}$ showing the unstable region, and (b) dimensionless global viscous growth rate $s^*$ (red curve) as function of the non-dimensional wavelength, showing the contribution of the different effects (constant non-dimensional inviscid growth rate: black curve, effect of long wavelength damping on the former: grey curve, and effect of viscosity on the former: blue curve) for the flowfield in figure 29.

Figure 31: Same configuration as in figure 27, but at two successive non-dimensional times (left: $t^* = 2.0$, right: $t^* = 5.2$), with the corresponding vorticity field in the cross-section. Scattered laser light allows visualizing the primary vortex even outside laser sheet plane (for $t^* = 5.2$).

Figure 32: Volume LIF visualizations for $Re_T = 5200$. Fluorescein (green dye) is injected on the ground to visualize the boundary layer and secondary vortex; Rhodamine (red dye) is painted on the flap tip to visualize the primary vortex (only the background one for visualization matters).

Figure 33: Same three-dimensional LIF visualizations as in figure 31, $Re_T = 2500$. Only the background vortex (red dye) is visualized, Fluorescein (green dye) is injected on the ground to visualize the dynamics of the boundary layer and detaching vortex sheets.

Figure 34: Front and side views of Crow instability development in the case of non-forced instability, $h_0 = 6b_0$, $Re_T = 3000$.

Figure 35: Crow instability amplitude obtained with forced perturbations (wavelength : $4.8b_0$ and amplitude : $0.04b_0$) showing the exponential growth (the logarithm being linear) once the instability is well developed.

Figure 36: Bottom and side views for $h_0 = 6b_0$ and $Re_T = 3000$. The Crow instability process is observed, as well as stretching and rebound of vortex rings.

Figure 37: Position of the vortices in the two planes of figure 10: (a) dimensional positions obtained from the visualizations; (b) symmetrised non-dimensional positions obtained by averaging the $y$-positions of the vortices of the pair and centring the $x$-positions on zero.

Figure 38: Bottom view at $t^* = 5.8$ for $h_0 = 6b_0$ and $Re_T = 4000$ showing the wavy deformation of the core rings resulting from the short wavelength elliptic instability.

Figure 39: Vortex spacing evolution versus time: (a) dimensional and (b) dimensionless parameters.

Figure 40: Vertical position of the vortices in the two planes and average vertical position.
1 Introduction

This report presents the work performed by CNRS-IRPHE in the framework of the European project FAR-Wake (Fundamental Research on Aircraft Wake Phenomena), Work package 3 (Wake evolution near the ground), Task 3.1 (Dynamics and decay in idealised conditions), Subtask 3.1.1 (Longitudinally uniform wakes).

The dynamics of a pair of counter-rotating vortices interacting with the ground is experimentally investigated. This flow is of primary interest in aeronautical applications. The hazard for following aircraft caused by the two strong counter-rotating vortices generated in the far wake of a leading aircraft is greatly enhanced during take-off and landing, where all aircraft use the same path. The first research programs launched in the 1970’s mainly in the US led to the definition of standard separation distances between aircraft, classified according to their Maximum Take-Off Weight. With the constant increase of air traffic and aircraft size, reduced separations between aircraft conserving the same level of safety are needed, implying a thorough understanding of wake vortex dynamics. One aim of the current research projects is to gain new knowledge on wake phenomena, necessary to develop strategies for wake alleviation and prediction, relevant for increasing safety and capacity of air transport.

In the present work, a temporally evolving pair of counter-rotating vortices in the vicinity of the ground is investigated experimentally. Laser-Induced Fluorescence (LIF) visualizations as well as Digital Particle Image Velocimetry (DPIV) measurements are carried out on longitudinally uniform vortex pairs generated inside a water tank. The aim of this qualitative and quantitative study, carried out on a relatively simple vortex pair configuration at moderate Reynolds numbers, is to gain new knowledge on the three-dimensional interactions of the vortices with the ground. In particular, the interactions between the newly formed secondary vortex and the primary one, as well as the instabilities developing and leading to the decay of the vortices are of primary interest. Six circulation-based Reynolds numbers in the range 1900–5500 were studied, in order to investigate the effect of this parameter on the overall dynamics. Most of the work concerns a configuration where the initial height of the vortices is equal to twice their separation distance, since it represents the situation encountered at take-off and landing of real aircraft. Some results for larger initial heights are also presented.

The report is organized as follows: in the following chapter, some theoretical considerations are recalled, and a synthesis of past work and present knowledge on the topic is given. The experimental details and methods used in this work are described in the chapter 3. Results are presented and analyzed in detail in chapter 4, followed by the conclusions.
2 Vortex pair dynamics

2.1 Basic considerations

Vortex structures are well-known to engineers and scientists because they play a key role in many base flows and engineering applications. Even non-specialists know conceptually what a vortex is. This is probably due to the fact that vortices are encountered in everyday life, from the simple bathtub vortex to tornados or aircraft wake vortices (figure 2), often visualized in the sky by condensation. However, no universal definition of a vortex really exists. In order to study vortex flows, vortices have been modelled in a more or less ideal way. One of the simplest models is the non-viscous Rankine vortex. It consists in the combination of a solid body rotation (azimuthal velocity linear with the radial position $r$) and an irrotational vortex (azimuthal velocity inversely proportional to $r$):

\[
\begin{aligned}
    u_\theta(r, \theta, z) &= \begin{cases} 
    \frac{\Gamma}{2\pi R^2} \cdot r, & r \leq R \\
    \frac{\Gamma}{2\pi r}, & r > R 
    \end{cases} ;
    u_r = u_z = 0 ;
\end{aligned}
\]

[1]

where:

- $\Gamma$ is the circulation (or strength) of the vortex defined as: $\Gamma = \oint_C \mathbf{u} \cdot d\mathbf{l}$, \hspace{1cm} [2]
- $C$ is a closed contour containing the vortex,
- $R$ is the vortex radius,

Another model, the Lamb-Oseen vortex, takes into account the viscous diffusion of vorticity:

\[
\begin{aligned}
    u_\theta &= \frac{\Gamma_0}{2\pi r} \left[ 1 - e^{-\frac{r^2}{24\nu t}} \right] 
\end{aligned}
\]

[3]

$v$ is the kinematic viscosity (about $10^{-6}$ m·s$^{-2}$ for water).

Lamb-Oseen vortices are Gaussian; i.e. characterized by a Gaussian distribution of vorticity. For a Gaussian vortex, the vortex core radius $r_c$ is defined as:

\[
r_c = r_{u_\theta = \text{max}(u_\theta)} = 1.12 \cdot a,
\]

[4]

where $a^2 = 4\nu t$. This model appears to be a good approximation of real structures. Leweke & Williamson (1998a, 1998b) showed that the vortices obtained in their experiments (using a similar set-up as in the present work – see section 3) are very close to Lamb-Oseen vortices when considering azimuthal velocity profiles.
The definition of a vortex is discussed in the first chapter of Greene’s book, *Fluid Vortices* (1995). The general (and ‘non-rigorous’) definition used in the latter for a vortex is the following: ‘a vortex is any region of concentrated vorticity [$\omega = \text{rot}(\mathbf{u})$]’. A huge amount of theoretical details (not repeated here) on vortices and vortex flows can be found in Greene (1995) and Saffman (1992).

In the present work, the dynamics of a pair of counter-rotating vortices evolving in the vicinity of the ground and in an otherwise irrotational flow is experimentally investigated. This base flow is fully characterized by a group of dimensionless parameters:

- The Reynolds number, usually based on the (absolute value of the) initial vortex strength $\Gamma_0$, which is the same for the two vortices, is defined as follows:
  \[ \text{Re}_r = \frac{\Gamma_0}{\nu}, \]
- The ratio of the initial cores radius over the initial vortex spacing: $\frac{r_0}{b_0}$, with $u_0(r_0) = \max(u_0)$,
- The initial distance from the ground non-dimensionalized as: $\frac{h_0}{b_0}$,
- The non-dimensional time (normalised by the convective time): $t^* = t \frac{\Gamma}{2\pi b^2}$,

Non-dimensional time is used in order to hide purely convective effects (i.e., a given feature appears at an earlier dimensional time at higher Reynolds number because the characteristic velocity at which the flow evolves is higher). In that way, Reynolds-dependent mechanisms may be investigated.

The following sub-sections present some of the significant earlier studies. The main results concerning the dynamics of a counter-rotating vortex pair out of ground effect (OGE) and in ground effect (IGE), as well as the remaining unanswered questions are presented. Investigations of wake vortex dynamics are numerous; although most of the studies relevant for the present work are presented, the following synthesis does not pretend to be exhaustive.

### 2.2 Dynamics “out of ground effect” (OGE)

The main features of the dynamics of a counter-rotating vortex pair are the instabilities that develop during their induced motion, eventually causing the decay of the pair. The dynamics of a pair of counter-rotating line vortices is determined by Biot-Savart induction. The velocity at a given point results from:

- The induction due to the other vortex. Considering two straight parallel line vortices of equal strength, this pair evolves with an uniform induced velocity:
\[ \nu = \frac{\Gamma}{2\pi b}, \]

where \( b \) is the vortex spacing, and

- The self-induced motion.

One instability of such a pair is the well-known long-wavelength Crow instability (figure 1), named after the author of the first three-dimensional stability analysis. Crow (1970) showed that a pair of uniform parallel counter-rotating vortices, without axial core flow and interacting in a non-stratified environment, is unstable. Symmetric wavy perturbations, whose axial wavelength is typically several times (about eight) the initial vortex separation distance, are amplified in planes inclined \( \sim 45^\circ \) with respect to the one containing the vortices. The stability of these perturbations is governed by the interaction of three effects on a given vortex, all of the same order of magnitude:

- The motion induced by the unperturbed other vortex, which corresponds, to first order, to a strain of maximum stretching in the \( 45^\circ \) direction (Widnall et al. 1974).
- The motion induced by the perturbation of the other vortex, whose angular dependence is more complicated.
- The self-induced motion. Kelvin (1880) showed that a wavy perturbation of a single vortex filament does not grow in time but rotates in a sense opposite of the vortex itself, which is a consequence of the vortex curvature imposed by the perturbation.

The Crow instability results from a balance between the stabilising effect of the self-induced rotation of the perturbation and the destabilising influence of the strain induced by the other vortex. When the circumferential component of the velocity field induced by the second vortex exactly cancels the self-induced rotation, the perturbation remains in a stationary plane and is amplified by the strain (induced by the second vortex) and leads to an exponential growth of its amplitude. The stationary plane is shown to lie \( \sim 45^\circ \) with respect to the line joining the two vortices and corresponds roughly to the direction of maximum stretching (figure 1). Widnall et al. (1974) described in details this mechanism and its relation to Kelvin’s modes. Leweke & Williamson (1998b) performed a careful comparison of their experimental measurements with analytical results. Crow instability is known to lead to the formation of periodic vortex rings after reconnection. Such a phenomenon is observed in the present work in the case of large initial heights (figure 36).

The second three-dimensional instability of a counter-rotating vortex pair is the so-called short-wavelength elliptic instability. Widnall et al. (1974) pointed out that some disturbance modes also had vanishing rotation rates for certain short wavelengths. They suggested that these modes are likely to lead to instability. Some subsequent stability analyses of vortex filaments in an externally imposed strain field confirmed this result (Moore & Saffman 1975; Tsai & Widnall 1976; Robinson & Saffman 1984). Short-
Figure 1: Crow instability observed in the wake of a transport aircraft showing the 45° plane of the long wavelength instability.

Wavelength instabilities were observed experimentally on vortex rings (Walker et al., 1987) and on vortex pairs (Thomas & Auerbach, 1994). Leweke & Williamson (1998a) investigated experimentally a pair of parallel counter-rotating vortices, uniform in the axial direction. They found that, at high enough Reynolds number (>2000), a short-wavelength instability (figure 3) appears and leads to internal deformations of the vortices. The origin of this deformation is explained by an elliptic instability of the vortex cores. This short-wavelength elliptic instability is shown to affect strongly the long-term evolution of the pair: the destruction of the vortex structures and the transition to a non-organized turbulent flow is greatly accelerated compared to the case of pure Crow instability.

Figure 2: Vortices in the wake of civil transport aircraft.

Figure 3: Long- and short- wavelength instabilities in a vortex pair $Re_p=2750$ (Leweke & Williamson, 1998a).
2.3 Dynamics “in ground effect” (IGE)

2.3.1 Two-dimensional interactions

The presence of the ground has two main effects on the vortex pair dynamics. The first one is due to an inviscid interaction, which can be modelled by two image (symmetric with respect to the ground line) vortices, whose strengths are equal and opposite to those of the real vortices. When the real vortices reach a distance from the ground equal to one half their initial separating distance (i.e. the distance between the vortex and its image is equal to the real separation distance between the two vortices), the interaction with the image vortices becomes greater than the interaction between the two real vortices. This effect causes the two vortices to move away from one another in a direction parallel to the wall (see the initial work of Lamb 1932, who showed that the inviscid trajectory of a point vortex in this case is a hyperbola, and the more recent review of Doligalski et al. 1994).

The second effect of the ground is of a viscous nature. One of the first relevant studies of the viscous interaction of a vortex pair with the ground is the experimental investigation of Harvey & Perry (1971). They examined the case of a two-dimensional vortex pair approaching a no-slip wall in a viscous fluid by studying flight test data from Dee & Nicholas (1968) and conducting wind tunnel tests. During their descent towards the wall, each primary vortex induces a cross flow at the ground and creates a boundary layer. Located beneath the primary vortex, the boundary layer is subject to an adverse pressure gradient that eventually causes it to separate from the ground. As it separates, the boundary layer rolls-up around the primary vortex into a secondary vortex which is of the opposite sign. This opposite-signed vorticity induces an upward velocity on the primary vortex and causes it to rise (rebound from the ground).

The basic dynamics of a vortex pair interaction with the ground in viscous fluids is a combination of these two motions: divergence of the two vortices along the wall and rebound from the wall.

This behaviour was confirmed by extended investigations. A large number of numerical studies were carried out. Peace & Riley (1983) numerically observed the rebound of a vortex pair approaching a free as well as a no-slip surface. Their explanation in terms of vorticity diffusion across the boundary layer was later supported by the analytical study of a vortex pair approaching a rigid surface, performed by Saffman (1991).

Türk et al. (1999) investigated numerically the effects of the Reynolds number on the vortex/ground interaction. For the large range of Reynolds numbers examined ($3\times10^2<\text{Re}_T<3\times10^6$), separation of the boundary layer occurred in all cases. Several newly formed vortices were observed as the Reynolds number increases. Minimum and maximum altitudes reached by the primary vortices for the different Reynolds numbers were also studied.

Puel & De Saint Victor (2000) simulated the unsteady two-dimensional interaction (before any sinusoidal perturbation develops) of a vortex pair with the ground in a laminar flow. Results are concordant with those of Türk et al. (1999): the separation of
the boundary layer as well as the formation of several successive vortices (more frequent as Re increases) is observed. The newly formed vortices are shown to slow down the lateral (inviscid) motion of the primary vortex and eventually cause it to develop loops. Successive rebounds (up to five at high Reynolds numbers) were observed.

A large number of numerical simulations have been performed in order to investigate the effects of additional phenomena (which are beyond the scope of the present work) on the vortex/ground interaction, such as crosswind (Robins & Delisi 1993; Corjon & Poinsot 1997), turbulence (Ash et al. 1994), stratified atmosphere (Delisi & Robins 2000). Increasingly complex vortex trajectories were observed as well as asymmetric features and multiple newly formed vortices.

### 2.3.2 Three-dimensional interactions

Three-dimensionality adds much more complexity to the interaction of a vortex pair with the ground and makes numerical simulations very costly. As a result, investigations of the three-dimensional problem were rare. It is only quite recently that it has received significant attention. With the improvements of computational capacities and numerical simulations (DNS, LES), interesting numerical investigations have been carried out since the end of the nineties. Papers relating experimental results are still relatively rare on this subject.

One of the first three-dimensional investigations is the experimental work of Ciffone & Pedley (1979). The effects of the ground on the trajectories and velocity profiles of aircraft wake vortices generated by towing models in a water basin in landing configuration were studied. They found out that the proximity of the ground causes significant modifications in the vortex trajectories, but did not alter the interactions and merging of the multiple wake vortices generated by the models. Deformations of the tangential velocity profiles, suggesting lower rolling moment, were also recorded while the maximum values were not affected. As the distance from the ground was reduced, a slower vortex descent, before spreading laterally, was observed. This 3D experiment tended to confirm the 2D viscous explanation for the vortex rebound (i.e., induced by boundary layer ejection and formation of a secondary vortex).

After this, the most significant investigations of three-dimensional interactions with the ground are mostly numerical simulations performed in the last decade. Luton & Ragab (1997) performed very interesting DNS simulations of an initially straight vortex pair interacting with the ground at $Re_f = 2196$. Three-dimensionality is shown to be introduced into the initially two-dimensional flow through a short-wavelength instability of the secondary vortices occurring after the primary vortex rebound. The mechanism is described in detail: a strong deformation of the secondary vortex and a distortion of the primary vortex at later stages are observed. Their description mentions a reconnection between the two secondary vortices and a partial reconnection between the secondary and the primary vortices before the secondary vortex breaks up due to stretching and viscous dissipation.
In a quite similar way, Corjon & Stoessel (1997) investigated the development of the Crow instability near the ground, as well as effect of cross winds, using DNS. Perturbations were imposed on the vortex pair in order to force Crow instability before interaction with the ground occurs. The Reynolds number was $Re_T = 5278$. The rebound phenomenon is shown to be similar to the two-dimensional case. The curvature of the primary vortex is transferred to the secondary one which is then wrapped around the primary vortex. A consequence is, according to the authors, a reconnection between the primary and the secondary vortices. The slowing of the separation of the vortices near the ground and the absence of classical (OGE) linking are in agreement with the flight experiment of Tombach et al. (1975). Unlike Tombach et al. (1975), however, no reconnection of the primary vortex with its image (also described as a reconnection with the ground) is observed. The simulations showed that the process leading to the bursting of the vortices IGE is mainly due to the viscous interaction, contrary to Tombach et al. (1975), who explained it in term of inviscid effects only.

In their Crow-like analysis, Kornev & Reichert (1997) found that for a vortex pair approaching the ground in an inviscid fluid, the Crow instability plane angle is modified as the initial height of the vortex pair is decreased. No clear explanation was given for this phenomenon. For initial heights lower than $1.7 b_0$, the growth rate is shown to decrease to zero at about $b_0/2$ before increasing again as the initial height is further reduced.

Proctor & Han (1999) carried out LES simulations of a landing observed at Dallas airport ($Re_T = 10^9$), in order to investigate the wake vortex decay IGE. Results are in good agreement with observations. Sensitivity to ambient turbulence and to wake generation height is studied. First, lateral and vertical transport is shown to be weakly influenced by ambient turbulence and may be well predicted with two-dimensional simulations. The rate of decay IGE is also weakly sensitive to ambient turbulence. It is not affected by the presence of the ground prior to the minimum altitude, but after this point the rate of decay is greatly enhanced. Wake vortex transport is affected by the ground up to $2.5 b_0$.

As a continuation of Proctor & Han (1999), Proctor et al. (2000) investigated wake vortex transport and decay IGE and the sensitivity to the initial values of circulation, height and vortex separation. The non-dimensional decay rate is found to be insensitive to these parameters; a simple decay relationship is given. Simulations showed good agreement with Proctor & Han (1999) concerning the strong vortex decay enhancement once the minimum altitude is reached. The phenomenon of ground linking (or reconnection with the image vortex), observed in real cases, is investigated. The influence of ambient turbulence is shown, no ground linking occurs for moderate to weak turbulence.

Paihlas et al. (2000, 2005) investigated experimentally mean velocity fields and turbulence occurring in the core of spatially-evolving counter-rotating vortex pairs IGE ($Re_T = 3.5\times10^4$). While vorticity distribution is concentric OGE, a strong deformation is observed IGE. Some modification in the streamwise velocity and Reynolds stresses are also observed. Kinetic energy in the cores is shown to be weakly affected IGE. Evidence of Crow and short wave-length instabilities in the main vortices was found.
Puel & de Saint Victor (2000) carried out three-dimensional steady space-developing computations. These computations were validated by comparison with the experimental results of Devenport et al. (1997), good agreement was achieved. Three-dimensional effects appeared mainly in the accounting of axial velocities. Although laminar flow was considered, turbulence is thought to be at the origin of the faster disappearance of the secondary vortices affecting greatly the primary vortex trajectory.

Finally, Fogg (2001) performed an experimental investigation of the interaction of a temporally-evolving pair of counter-rotating vortices (no axial flow) with the ground. Similar to the present work, the development of Crow instability IGE as well as initial height effects are studied using Laser Induced Visualizations (LIF) and Digital Particle Image Velocimetry (DPIV) in a water tank at low Reynolds number ($Re_\Gamma = 1020$). Two-dimensional dynamics IGE are found in good agreement with Harvey & Perry (1971) and Türk et al. (1999). The study of Crow instability development in the vicinity of the ground showed a rotation of the instability plane and a strong difference in the dynamics of the two cross-sections peak plane (plane in which the two vortices move closer) and trough plane (plane in which the two vortices move way from one another). The perturbation amplitude departs from Crow’s exponential growth as the secondary vortex is generated. The dynamics of Crow instability depend on the initial height. An investigation of initial height effects is performed: for low enough $h_0$, reconnection does not occur. It is important to notice that all quantitative results strongly depend on the initial wavelength and amplitude of the forced long-wavelength instability; among others, the critical initial height below which reconnection does not occur.

As for the two-dimensional case, numerous studies investigated the interaction with the ground with additional effects such as stratification (Hamilton & Proctor 2000; Proctor & Switzer 2000), ambient turbulence (Proctor & Switzer 2000) or convection (Holzäpfel et al. 2000). To conclude with this section, it is worth mentioning the results obtained for a vortex ring impacting a plane wall. Even though the geometry is different, the study of such a flow has a double interest here. First, because it presents some interesting features similar to the ones observed with a vortex pair: boundary layer creation and ejection, newly formed vortex rings, rebounds, loops trajectory, short wavelength instabilities... Second, because at the late stages of Crow instability development, periodic vortex rings are obtained from the initial vortex pair and one can imagine them interacting with a plane wall. The main difference between the dynamics of a vortex pair and the one of a vortex ring is the effect of intense stretching in the latter. Walker et al. (1987) performed a very careful study of a vortex ring impacting a wall. Theoretical, experimental and numerical results are compared for Reynolds numbers based on the ring diameter between 105 and 3000. The phenomenon of rebound, described in detail, is similar to the case of a vortex pair: the unsteady boundary layer created at the wall as the ring reaches a height of about one diameter is ejected, and a secondary vortex ring of larger diameter is created causing the rebound. In most cases, a tertiary vortex ring was observed. After the secondary ring was ejected, short wavelength instabilities were observed and led to a highly turbulent flow. This work has been supported by the subsequent numerical study of Orlandi & Verzicco (1993).
2.4 Objectives of the present work

The experiments carried out in the present work are similar to that of Leweke & Williamson (1998a,b) for a vortex pair OGE, and Fogg (2001) for a vortex pair IGE. The dynamics of a counter-rotating vortex pair approaching the ground in an otherwise quiescent fluid is to be studied. The vortex pair is evolving without axial flow (contrary to real aircraft trailing vortices). The vortices are initially laminar and uniform in their axial direction. Once the vortices are generated, each vortex induces a velocity on the other one which results in a translation of the vortex pair towards the ground.

This work may be seen as a continuation of Fogg’s (2001) study. Careful flow field visualizations and measurements are performed, in order to characterize in detail the ground interactions. The following remaining questions on such interactions are of particular interest. The interactions of an initially two-dimensional (straight) vortex pair with the ground and the mechanism leading to the introduction of three-dimensionality through the secondary vortices are investigated and compared to numerical results. The dynamics of the secondary vortex is of crucial interest since it greatly affects the dynamics of the primary one, making the flow much more complex than in the two-dimensional case. This work extends Fogg’s results (obtained for $Re_f = 1020$) to a wider range of Reynolds numbers (1900 to 5500). Analytical work is also performed to support experimental observations. The following section describes in detail the experimental set-up and techniques used in this study.
3 Experimental details

3.1 Experimental set-up

Experiments were carried out in a Plexiglas water tank (135×56×56 cm$^3$) shown in figure 4. The set-up is the same as the one used by Meunier (2001) to investigate the merging of two vertical co-rotating vortices. The vortex generator has been modified in order to generate two counter-rotating vortices. This set-up is very similar to the one used by Leweke & Williamson (1998a,b) and by Fogg (2001). Though the main parts of the apparatus existed, its adaptation to the present work was time-consuming. In particular, the generation of well-formed vortices has required numerous experiments. The time necessary to carry out such rigorous experiments may explain the relatively few experimental results existing on vortex dynamics.

Figure 4: Experimental apparatus.
The vortex generator consists of two flat aluminium plates, anodized in order to avoid oxidation, and painted in black for visualization matters. The plates are fixed vertically to a common base and their free edges are sharpened to an angle of 30°. The vortex generator is fixed on a carriage translating vertically in order to be able to remove it from the water. Its high aspect ratio is necessary to minimize end- and air/water interface-effects to obtain a homogeneous flow in the central part of the tank. Another device used to minimize the influence of end effects is a flat plate positioned closely beneath the vertical vortex generator’s plates (figure 5). The maximum efficiency is obtained when the sharp edge of this device links the two sharp edges of the vortex generator. In that case, the vortices generated by the two flaps are linked at the bottom by the vortex generated at the edge of this flat plate. This configuration prevents the tornado effect observed when a vortex ends at a wall (bottom of the tank in our case) and leading to its rapid decay.

![Figure 5: Bottom of the water tank showing the devices used to avoid end effects and to position the vertical plate simulating the ground.](image)

The symmetric inward motion of the plates is generated by a computer-controlled step motor connected to the plates by the mechanical system presented in figure 6. The ground is simulated by a vertical (orthogonal to the induced motion of the vortex pair) Plexiglas plate, with a varying distance from the vortex generator in order to investigate effects of initial height $h_0$. 

17 / 60
The motion history of the plates, shown in figure 8 and based on Leweke & Williamson (1998b), has been experimentally optimized in order to generate uniform vortex pairs without undesirable perturbations. The angular velocity is obtained from the following analytical function (see figure 7 for a definition of the geometry):

\[
\frac{\partial \theta}{\partial t} = \frac{V_{\text{max}}}{0.1297} \cdot \frac{d \cdot r_0}{B^3 \cdot \alpha} \cdot \frac{1}{(\theta - \theta_i)^2} \cdot \left[1 - \exp\left(-\frac{B \cdot \alpha \cdot (\theta - \theta_i)^2}{r_0}\right)^{\frac{3}{2}}\right]
\]

where:

- \( r_0 = 0.25 \, \text{cm}, \)
- \( \alpha = 0.7, \)
- \( \theta_i = \theta_f - \sin^{-1}\left(\frac{S - b}{2B\alpha}\right) \) is the initial angle of the plates and \( \theta_f = 12^\circ \) is the final one,
- \( b = 2.5 \, \text{cm} \) is the initial vortex spacing desired,
- \( V_{\text{max}} \): maximum mean outflow velocity varied according to the desired Reynolds number, and obtained by dividing the volume swept per unit of time by the instantaneous distance between the plates: \( V_f = \frac{B^2 \cdot \theta}{d} \),
- \( d = S - 2 \times B \times \sin(\theta_f) \) is the instantaneous distance between the flap tips,

The variation of \( V_f \) obtained consists in smooth acceleration and deceleration in order to generate uniform laminar vortices. The deceleration is important since too strong decelerations generate opposite-signed vorticity, hence instabilities in the initial vortex pair. The motion is stopped at an angle of \( 12^\circ \), before the plates touch, to avoid the strong thin jet occurring when the distance between the plates decreases to zero. The
time of the motion is long enough so that the vortex pair has moved down and is not affected by the counter-rotating stopping vortices generated at the edges of the flaps when the motion ends. When the flaps are impulsiively closed from an initially almost parallel position, the sharp edges generate vorticity that rolls up into two counter-rotating vortices. During the motion of the plates, circulation is continually added to each vortex resulting in a decrease of the vortex spacing. When the motion of the plates stops, no circulation is added anymore and the two vortices evolve freely. It has been observed that the end of the plate motion corresponds, in the absence of ground effect, to the time at which the trajectories of the vortices are parallel (i.e. the vortex spacing $b_0$ and the induced velocity are constant). All initial heights studied in this work are large enough ($>2b_0$) to avoid the vortex generation to be affected by ground effects.

Finally, much care has been taken in the alignment of the whole set-up to achieve the symmetry of the plate motion. Much time has also been taken to optimize the experimental procedure; i.e., the way the dye is painted on the plates, the quantity of dye, the speed of the translating chariot descent in the water in order to minimize the generation of turbulence and dye diffusion in the tank, and obviously, the plate motion. In ideal conditions (i.e., perfect symmetry of the set-up and absence of residual fluid motion in the water tank), the dynamics of this base flow would be perfectly symmetric. Any asymmetry found in the results (trajectories, vortex strength...) are due to experimental uncertainties (set-up, visualization and measurements techniques) or to the development of instabilities. A detailed description of the two experimental techniques used in this investigation to characterise the vortex flow is given in the following subsections.
3.2 Laser-Induced Fluorescence technique

Laser-induced fluorescence visualizations are performed to obtain qualitative information on the 2D and 3D dynamics of the pair. Fluorescein dye is painted on the edge of both flaps prior to introduction of the apparatus in the water. When the plates are impulsively closed, the fluid ejected from the volume between the plates picks up the dye which rolls up into the initial vortices. Vortex cores are then visualized by illuminating with a 5W Argon laser. Images are recorded on standard video tape using a 720×576 pixel CCD camera.

Two different types of visualization are carried out. The first one consists of painting dye all along the plates, then illuminating the whole tank with a light cone, in order to visualize in volume the evolution of the vortex pair. These visualizations are recorded using two cameras: one for a side view (view parallel to the ground and perpendicular to the vortex axis), the second one for a front view (from below the ground in the direction of the pair motion). The second type of visualizations is obtained by illuminating a cross-section of the vortex pair with a laser sheet. An example of the images recorded is shown on figure 9b. By using two parallel light sheets, one in the peak plane (i.e., of maximum outward displacement, see figure 10), and one in the trough plane (i.e., of maximum inward displacement), as done by Fogg (2001), the dynamics of Crow instability can also be studied (figure 9a). To position the two planes, the phase distribution of the instability along the vortices has to be known in advance. For this purpose, sinusoidal waves have been added onto the vortex generator plate tips, following Leweke & Williamson (1998b). This was achieved by fixing thin plastic sheets at the flaps tips. The perturbations used are characterized by a wavelength of about 12 cm (≈ 4.8b₀ in the present study) and an amplitude of 1 mm (≈ 0.04b₀). It should be noted that the actual amplitude of the Crow instability at the initial time t₀ (i.e., after vortex generation) does not necessarily have the same value. It needs to be determined from measurements, in order to characterize correctly the initial conditions.
Some of the parameters characterising may be estimated from these cross-section visualizations. The instantaneous positions of the vortex cores, directly obtained from the visualizations, give the vortex spacing, velocity displacement $V$ of the vortex pair and the distance from the ground. The Reynolds number can then be estimated using:

$$Re = \frac{\Gamma}{\nu} = \frac{2\pi \times b_0 \times V_0}{\nu}$$

where $V_0$ is the initial induced velocity of the vortex pair, and $\Gamma_0$ is the initial circulation of the vortices obtained from equation [5],

The same parameters may be obtained from PIV measurements. Comparison between the two sets of results shows good agreement. For low initial heights, however, the velocity displacement may be affected by the presence of the ground, leading to underestimated Reynolds numbers. PIV measurements give accurate results for all initial heights since the vortex circulation generated by the vortex generator is not affected by the proximity of the ground.
3.3 Particle Image Velocimetry

Two-dimensional PIV is carried out in order to obtain quantitative measurements of the velocity fields. The measurement set-up uses laser sheets as described above and shown on figure 10. The flow is seeded with spherical plastic particles (Optimage Ltd., UK) of density 1.03 g/cm$^3$ and mean diameter 30 μm. The measurement plane is illuminated with a laser sheet of 3-5 mm thickness using an Nd-YAG Quanta-Ray PIV-200-10 pulsed laser from Spectra-Physics. The frequency of the pulses is 10Hz and the power of each pulse is approximately 160 mJ. Images pairs are recorded with a high-resolution (1008x1018 pixels) CCD Kodak Megaplus ES 1.0 digital video camera synchronized with the laser pulses. The system is externally triggered by the high quality delay generator Stanford Research Systems DG 535.

Velocities are obtained from a cross-correlation algorithm, using software (DPIVsoft) developed at IRPHE under the MATLAB environment. The interrogation windows, of dimensions 32×32 pixels, are translated and deformed according to the velocity gradients to keep the height of the correlation peaks as large as possible in case of high velocity gradients. A field of 60×60 vectors is obtained, to which a median filter (introduced by Westerweel 1994) is applied in order to detect and correct spurious vectors. Once the velocity field is obtained, two additional median filters are applied in order to optimize the final velocity field and the computation of the vorticity field. Technical details concerning the cross-correlation algorithm can be found in Meunier & Leweke (2003). In the next section, the initial conditions of the flow under consideration are given.
3.4 Initial flow field

Once the experimental set-up is operational, it is necessary to determine the parameters defining the initial conditions. The first step is to define the initial time \((t_0\) or \(t^*_0\)). In order to enable subsequent comparable numerical simulations, it is only necessary to determine all the parameters at a given and well-defined initial time \(t_0\), whatever it is. From a physical point of view (in order to investigate ground effects), it appears logical to define the initial conditions at a time \(t_0\) at which the vortices are OGE.

In the following sections two different configurations are experimentally investigated. For the study at large initial height the initial time is defined as the time from which the vortices show constant vortex spacing and induced velocity. It appears in the experiments that it corresponds to the time at which the flap motion ends, when no more circulation is added. The main part of the present work concerns the interactions of a pair of uniform vortices with the ground (no forced perturbations). This implies that the vortex generation occurs at a low enough distance from the ground in order to avoid the development of Crow instability. For this configuration, the initial time is defined as the time at which \(h_0 = 2b_0\). The vortices remain uniform along their longitudinal axes (no development of Crow instability) and the dynamics of the pair is only weakly affected by ground effects at this initial time. The initial parameters are determined using PIV measurements (figure 11). This is meaningful since at \(t_0\) (i.e., at \(h_0 = 2b_0\)), the flow is still two-dimensional.

![Figure 11: Initial \((t^* = 0)\) vorticity field \([s^{-1}]\) obtained from PIV measurements for \(Re_r = 5200\).](image)

Figure 11 shows the initial vorticity field computed from the velocity field measured by PIV for \(Re_r = 5200\). The two zones of vorticity above each vortex correspond to the opposite-signed vorticity generated by the deceleration and end of motion of the flaps. One can notice that vorticity is also generated at the wall, as expected.
Two-dimensional PIV measurements allow determining the characteristics of the initial vortices. Figure 12 shows the azimuthal velocity and circulation with respect to the radial position for one of the two vortices. Experimental data obtained from PIV measurements fit very well with the theoretical profiles of a Gaussian or Lamb-Oseen vortex (eq. 2 p.10), as found by Leweke & Willimanson (1998, 2004) and Fogg (2001) with similar apparatus. This result shows that the vortices studied in the present work may be considered as Gaussian with very good accuracy. This will allow performing analytical work by modeling the vortices by two Lamb-Oseen ones.

![Radial profiles of azimuthally averaged swirl velocity and circulation](image)

**Figure 12**: Radial profiles of azimuthally averaged swirl velocity and circulation ($v_\theta = 2\pi r V_\theta$), for $Re_\Gamma = 5200$. Symbols represent measurements from PIV, solid lines are least-squares fits to the theoretical profiles of a Lamb-Oseen vortex (fit parameters: $\Gamma = 50.8 \text{ cm}^2/\text{s}$, $a = 0.433 \text{ cm}$). The dotted red line represents $r = b_0/2$.

Six different Reynolds numbers were investigated: $Re_\Gamma = 1900$, 2500, 3500, 4100, 5200 and 5500. It was observed that, with the experimental set-up described above, the initial vortex spacing is slightly dependant on the Reynolds number. It was found to increase slightly and quasi-linearly with the Reynolds number (figure 13). A variation of less than 15% of the average initial vortex spacing ($b_0 = 2.5 \text{ cm}$) is obtained.

By analyzing successive PIV measurements, initial parameters as well as their evolution in time are obtained. Figure 14 shows the results obtained for $Re_\Gamma = 5200$. Typical evolution of the distance from the ground $h$ and of the vortex spacing $b_0$ are shown. The vortex core radius is compared to that of a Lamb-Oseen vortex, i.e., $r_c^2 = 1.12^2 \times 4 \nu t$, and shows very good agreement with the latter. The vortex core radius is, on average, $r_{c0} = 0.21 b_0$.

In the next section, results are presented and discussed in detail. The main body of this work concerns the interactions of a pair of uniform counter-rotating vortices with the ground.
Figure 13: Dependence of the initial vortex spacing on the initial Reynolds number.

Figure 14: Evolution of vortex parameters for $Re = 5200$. (a) dimensional units, (b) non-dimensionalised with $b_0$, the theoretical relation $r_c = 1.12a$ for a Gaussian vortex has been used.

$h$: mean distance between the vortices and the wall; $r_c$: mean core radius; $a$: mean core radius for a Gaussian vortex; $\Gamma$: mean circulation. The black dotted line corresponds to the theoretical core growth of a Lamb-Oseen vortex.
4 Results

4.1 Two-dimensional dynamics

In this and the following section, the initial height of the vortex pair is always $h_0 = 2 b_0$. Larger initial heights will be considered in section 4.3. As seen in the previous chapter, LIF visualisations and PIV measurements are used to investigate the two-dimensional dynamics. Figures 15 and 16 show axial vorticity fields and cross-section dye visualizations at the same successive non-dimensional times for $Re_\Gamma = 5200$. As time increases, the vortex pair approaches the ground. The vorticity generated at the wall and the boundary layer thickness increase. At $t^* = 1.8$, the boundary layer starts detaching. At $t^* = 3.5$, a vortex sheet is wrapping around the primary vortex. This opposite-signed vorticity rolls up into a secondary vortex. The primary and secondary vortices form a dipole which has an upward induced velocity that causes the rebound of the primary vortex. As the secondary vortex wraps around the primary one, the induced velocity affects the primary vortex trajectory (rebound and loops, figure 17).

At $t^* = 4.9$, a tertiary vortex is formed. At $t^* = 6$, the bursting of the vortex system occurs. The mechanisms observed here for the two-dimensional dynamics are in agreement with previous investigations (see section 2.3.1). Although dye has been painted on the flaps only, the visualizations at times $t^* = 3.5$, 4 and 4.9 make the secondary vortices visible. Yet one can notice that the flow is symmetrical (as expected in ideal conditions) at $t^* = 3.5$, while it becomes non-symmetrical as time goes on ($t^* = 4$ and 4.9). This point will be developed later.
Figure 15: Vorticity fields for $Re\gamma = 5200$ and $h_0 = 2b_0$ of one vortex with corresponding cross-cut visualisations at successive non-dimensional times.
The vortex trajectories presented figure 17 obtained from cross-cut visualizations show vortex rebounds as well as loop trajectories. While the non-dimensional lateral transport, as well as the maximum and minimum rebounds of the vortices, show weak sensitivity to the Reynolds number, a characteristic evolution of the dimensionless trajectory with respect to the Reynolds number clearly appears. The main Reynolds number-dependent feature is the shape of the first loop. At low Reynolds numbers, this first loop is small and followed by smaller and smaller successive loops. At higher Reynolds numbers, only one loop of large amplitude occurs before the breakdown of the vortices.
Figure 17: Trajectories for three different Reynolds numbers. Figures on the right side show non-dimensional and symmetrised trajectories where $x/b_0 = 0$ corresponds to the symmetry axis and the $y/b_0$ values are obtained by averaging the $y$-position of the two primary vortices.
An interesting observation concerns the symmetry of the dimensional plots (left column on figure 17). Before the minimum altitude is reached, the symmetry of the trajectories is relatively good. As the vortex rebounds, the symmetry is lost. The lack of symmetry observed on the plots obtained from LIF may be explained in two different ways. One reason may be linked to the method used to determine the vortex centres (directly from the images, “by hand”). As time goes on, the dye diffuses due to viscosity, and the primary vortices dissipate during their interaction with the secondary ones, making the determination of the vortex centres less accurate at late times. To avoid this problem, the vortex centres can be determined from the velocity fields obtained from PIV: the vortex centres correspond to the position of maximum (positive) and minimum (negative) vorticity.

Figure 19 shows the trajectories obtained for one (maximum vorticity) of the two vortices of the pair. Although these results are obtained from a different experiment by using a different method (PIV measurement focused on one vortex to improve resolution), very good agreement with visualization results are observed. The same weak sensitivity to the Reynolds number appears for the non-dimensional lateral transport and maximum and minimum altitudes reached during rebound. The Reynolds-dependence of the shape of the first loop is also observed. The main difference is the maximum non-dimensional time up to which the vortex centres are plotted. This maximal value of $t^*$ corresponds to the last time before an important scattering occurs in the determination of the vortex centre (i.e., of the point of maximum vorticity). This scattering is due to the bursting of the primary vortex into small-scale structures (at $t^* = 6$ in figure 16). This maximum time is Reynolds-dependent. The values of this parameters are presented figure 18 for the three Reynolds numbers considered here. As the Reynolds number increases, the maximum value of $t^*$ before vortex bursting decreases significantly.

![Figure 18: Maximum non-dimensional time before bursting of the primary vortex, as function of Reynolds number.](image-url)
Recalling the asymmetry in the vortex trajectories obtained from dye visualizations: the first explanation was the accuracy of the vortex centres determination due to vortex bursting and dye diffusion. The second hypothesis is that the flow becomes intrinsically asymmetric as a consequence of the interactions between the primary and the secondary
Figure 20: Vertical position of the primary vortices. $t^* = 0$ is defined as the time at which $h_0 = 2b_0$.

vortices. Figure 20 shows the evolution of the non-dimensional distance from the ground with respect to the non-dimensional time obtained from dye-visualizations. A quasi-symmetry between the two primary vortices is observed until $t^*$ reaches a value between 3 and 3.5 (corresponding to the maximum rebound of the symmetrised vortices). As the primary vortex goes down (due to the relative position of the secondary vortex) after the first rebound, an asymmetry between the two primary vortices appears clearly. At this time ($t^* = 3.5$), the vorticity fields show features close to configurations in which three-dimensional instabilities develop.
In order to conclude with the two-dimensional dynamics, some general results relevant for the aeronautical application are presented below. One of them is the evolution of the circulation in a cross-section. This is of particular interest since one of the key parameter is the time during which the wake is “strong” enough (i.e., vortices of high circulation) to cause hazard.

Figure 21 shows the circulations computed for a domain delimited by the symmetry axis and the ground, hence including one primary vortex only. The domain used for the calculation is the same as the one shown for the vorticity fields (figures 15 and 16). The circulations are computed from the velocity fields using equation [2] (page 8). For the primary vortex circulation, a closed contour is defined ‘by hand’ in order to include the primary circulation only (not any opposite-signed circulation generated at the ground). For the total half-domain, the circulation is computed along the symmetry axis, the ground line (zero velocity) and the lateral and top borders of the domain, in order to close the contour. The third circulation is computed by taking the difference between the primary circulation and the total half-domain circulation. It corresponds to the circulation of the vorticity generated at the wall (boundary layer, secondary vortices...).
At $t^* = 0$, after the phase of vortex generation, the circulation of the primary vortex is at its maximum. Until $t^* = 1.5$, this value is approximately constant: the primary vortex is not affected by the ground and its decay is very weak. For the total half domain however, the decay starts from $t^* = 0.3$, after a short period of more or less constant circulation. This decay is due to the increase of vorticity (circulation opposite to that of the primary vortex, represented by the squares) generated at the ground as the primary vortex approaches the ground. As time goes on (and as the primary vortex is approaching the ground), the vorticity generated at the wall diffuses by viscous effect, the boundary layer, hence the opposite-signed vorticity, increases. Until $t^* = 2$, the decay of the primary vortex is weak while the total circulation has significantly decreased due to the increase of opposite-signed circulation. From $t^* = 2$, the primary vortex starts decaying while the opposite-signed circulation still increases. As a result, the total circulation decreases significantly until $t^* = 3.5$.

The table below sums up some of the results provided by figure 21. Non-dimensional times above 5 are not considered since the three-dimensional investigation presented in the next section shows that the dynamics is affected, for latter times, by an axial flow generated at the bottom of the tank.

<table>
<thead>
<tr>
<th>$t^*$</th>
<th>$\Gamma_{\text{primary vortex}} / \Gamma_0$</th>
<th>$\Gamma_{\text{primary vortex}} - \Gamma_{\text{total}} / \Gamma_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>0.19</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0.99</td>
</tr>
<tr>
<td>2</td>
<td>0.97</td>
<td>0.70</td>
</tr>
<tr>
<td>3</td>
<td>0.92</td>
<td>0.71</td>
</tr>
<tr>
<td>4</td>
<td>0.89</td>
<td>0.71</td>
</tr>
<tr>
<td>5</td>
<td>0.71</td>
<td>0.71</td>
</tr>
</tbody>
</table>

Figure 22: Average (over the three Reynolds numbers) circulations of the primary vortex and opposite-signed vorticity at different non-dimensional times. $\Gamma_0$ is the initial ($h_0 = 2b_0$) circulation of the primary vortex.

At $t^* = 5$, although the total circulation is low (about $0.19\Gamma_0$), the circulation of the primary vortex and opposite-signed vorticity are relatively high. Figure 23 illustrates the high level of shear in some regions of the flow. Such circulation and shear may impose strong (and hazardous) rolling moment and stress to any object (aircraft...) going through this velocity field.

The characteristic dipole structure of the vorticity distribution after rebound (figure 23) is likely to lead to three-dimensional instabilities. Experimental results concerning the three-dimensional dynamics of vortex pairs in ground effect are presented in the following section.
Figure 23: (a) Vorticity field for $Re_f = 5200$ at $t^* = 3.6$ showing the line joining the vortex centres. (b) Azimuthal (normal to the line) velocity [cm/s] as function of the position [cm] along this line. The origin is the middle point between the two vortex centres.
4.2 Three-dimensional dynamics

Volume LIF visualizations were performed to obtain qualitative information of the three-dimensional dynamics. Two-dimensional velocity fields obtained from PIV measurements may be then used to carry out analytical work, in order to validate the hypothesis. Comparison with Direct Numerical Simulation (DNS) performed at UCL is also presented.

The two-dimensional investigation (previous section) showed that as the primary vortices approach the ground, wall vorticity detaches from the ground into vortex sheets that roll up. Secondary opposite-signed vortices are generated and orbit around the primary ones. This is in agreement with previous investigations. Direct Numerical Simulations have been performed at UCL to investigate the three-dimensional interactions between the primary and secondary vortices (Duponcheel et al 2006).

A visualization of the dynamics of a vortex pair in ground effect obtained from these simulations is presented in figure 25. Iso-contours of vorticity show that until $t^* = 2$ (corresponding to $t^* = 3$ in the present experiments since $h_0 = 2b_0$ while $h_0 = b_0$ in the simulations), the flow is uniform in the axial direction of the vortices. The same two-dimensional dynamics as described above are observed.

At later times (from $t^* = 2.63$), a short wavelength instability develops on the secondary vortex and the rolling up of this wavy vortex leads to a complex three-dimensional picture (see $t^* = 3.77$). A close-up view of this phenomenon, obtained at later time with a larger longitudinal domain, is presented figure 24. It appears clearly that the instability of the secondary vortex combined with its wrapping around the primary one, results in a characteristic feature: vortex streaks perpendicular to the primary one.

![Figure 24: Iso-contours of vorticity obtained by DNS (Duponcheel et al. 2006) at $t^* = 3.96$ for $Re_f = 5000$. The initial height is $b_0$, the longitudinal extent of the computational domain is $4b_0$ and the initial perturbations are white noise (random and very small amplitude).](image)
Figure 25: Same configuration as shown on figure 24, except with a longitudinal extent of $2b_0$:
Evolution of the vortex system in the vicinity of the ground (Duponcheel et al. 2005).

Figure 26 shows experimental results obtained from volume LIF. Side view visualizations of one vortex only (i.e. no dye has been painted on the second plate in order to visualize the dynamics with precision, without any superposition of the two vortices) are compared for three different Reynolds numbers and for successive non-dimensional times. Since the visualized vortex is the one furthers away, the secondary vortex (not visualized) starts wrapping around behind it in this view.
Figure 26: Side view visualizations, only the farther vortex is visualized. The first row corresponds to the initial time $t_0$ ($h_0 = 2b_0$), the second row to the minimum distance from the ground ($t^* = 2$), the third row to the maximum rebound ($t^* = 4.5$), and the last row ($t^* = 6.4$) shows a later time. The top of each image corresponds to the plate level, the bottom to the ground.
As a consequence of the experimental procedure, the dye is not only concentrated in the vortex core but allows visualizing an extended radius that shows the development of a short-wavelength instability.

Figure 26 shows different interesting features. First, the dynamics appears to be quite strongly Reynolds number-dependent. In general, however, one can notice that a first instability appears on the top (periphery) of the primary vortex ($Re_T = 3500$, $t^* = 4.5$) and then, affects the whole periphery of the vortex ($Re_T = 5200$, $t^* = 2$). As the primary vortex rebounds and interacts with the secondary one, a second instability appears at the periphery of the primary vortex ($Re_T = 1900$ and $3500$, $t^* = 6.4$). The latter instability, of larger wavelength (about twice the first one), also affects the primary vortex core. Finally, the whole process leads to a rapid decay of the vortex system and the bursting into small-scale structures. The two instabilities affecting the vortex are characterized by short wavelengths. The second one seems to correspond to the instability observed (first, on the secondary vortex, and then on the primary one) in the simulations (figure 24 and 25). This instability develops initially inside the secondary vortex; then, as this perturbed vortex wraps around the primary one, a resulting instability of similar wavelength develops on the primary vortex. The first instability, affecting the vortex periphery only, has a very short wavelength (about half the one of the second one) and has never been reported in previous studies dealing with wake vortex pairs in ground effect. A cross-section visualization of the primary vortex showing this short wavelength instability at its periphery is presented in figure 27.

Figure 27: Longitudinal cross-section of one of the two primary vortices, showing the short wavelength centrifugal-type instability developing at its periphery. The laser sheet used for the LIF visualization is parallel to the ground and positioned at about $h = 0.6b_0$ (height of primary vortex minimum rebound, i.e. $t^* = 2$). The secondary vortex is not visualized. $Re_T = 5200$. 

---

39 / 60
The figure above shows features very similar to those observed in the case of centrifugal-type instabilities. Such instabilities have been observed by Thompson, Leweke & Hourigan (2005) in their study of the flow generated by the impact of a sphere on a solid wall; and by Billant, Colette & Chomaz (2003) while investigating a counter-rotating vertical vortex pair in a stratified and rotating fluid. Centrifugal instabilities are observed in axisymmetric flows when the vorticity changes sign (region of decreasing circulation) in some outward region of the flow. Details about the development of this instability in the present case will be presented in the next subsection. Going back to figure 26, general dynamics have been described above, but a strong Reynolds dependence was observed. At \( Re = 1900 \), the centrifugal-like instability does not develop (or at least, only very slightly at \( t^* = 4.5 \)). Then, at \( t^* = 6.4 \), the second instability has developed and affects the primary vortex core. This instability, whose wavelength is of the same order of magnitude as the one observed in the simulations, is thought to result from the interactions with the orbiting secondary vortex. A simplified sketch of the process leading to the perturbation of the primary vortex core is presented figure 28. Experimental and numerical work seem to be in good agreement on this point.

![Figure 28: Simplified sketch of a longitudinal section of the primary vortex at late times showing cross-sections of the secondary vortex and the resulting deformation of the primary one.](image)

As the Reynolds number increases, the first centrifugal-type instability occurs sooner when considering non-dimensional times. The development of the two instabilities is stronger and the decay of the primary vortex (and of the whole vortex system) is greatly enhanced. At this point, it is important to notice that at \( t^* = 6.4 \) (\( Re = 5200 \)), axial flow rising from the bottom of the water tank (from right to left on figure 26) starts affecting the dynamics of the vortex system. This axial flow is generated by the experimental setup (end-effect at the bottom of the tank). As a result, later times are not investigated and at \( t^* = 6.4 \), the decay may be slightly enhanced by this phenomenon. The following subsections present a detailed analysis of the two instabilities observed here.
4.2.1 Centrifugal-type instability

The origin of the first instability has to be determined, since it has never been reported (to the knowledge of the authors) in previous investigations of wake vortex IGE. An experiment similar to that presented on figure 27 ($Re_r = 5200$), except that the artificial ground was removed from the tank, has been carried out. The same type of instability has been observed, suggesting that it may be a consequence of the vortex generation (experimental procedure) rather than a result of the interactions with the ground. Although no paper about wake vortex dynamics relates such instabilities, Leweke & Williamson (1998a,b) encountered similar features in some of their experiments (using the same vortex generator), but did not consider and investigate this phenomenon further.

The size of the characteristic structures of the instability observed (figure 27) makes measurements and simulations very difficult, due to the resolution needed. It is believed that the vorticity of these structures is extremely weak compared to that of the primary and secondary vortices, hence they would hardly affect the overall decay, but no quantitative data is available. However, this first instability may influence the wavelength and development of the second one which has been observed in the simulation and may be responsible for the rapid decay of the vortex system.

A generalized theory of centrifugal instability in two-dimensional flows with closed streamlines, and in particular in axisymmetric flows, has been presented by Bayly (1988). Thompson, Leweke & Hourigan (2007) applied Bayly’s theory to obtain a theoretical prediction concerning centrifugal instability in the impacting sphere flow and found a good agreement with their experimental and numerical results. In the present work, this theory is applied to support the hypothesis of a centrifugal-type instability.

The theory is applied to the two-dimensional flow field presented figure 29. At $t^* = 1.2$, the instability is developing and the growth rate is likely to be close to its maximal value. According to Bayly’s theory for general inviscid two-dimensional flows,
A sufficient condition for centrifugal instability is that the streamlines be convex closed curves, with the magnitude of the circulation decreasing when going outward in some region of the flow. Figure 29 shows that at $t^* = 1.2$, these conditions are broadly satisfied. A small region of decreasing circulation is observed at a radial position of about $b_0/2$. The primary vortex is assumed to be axisymmetric (which is not exactly true, but valid to a good approximation). Azimuthal velocity and circulation are averaged over $\theta$ (in a local polar coordinate system centred on the primary vortex centre, figure 29b), in order to apply the axisymmetric theory. Using the azimuthally averaged values also allows elimination of measurement uncertainties and noise. In Bayly’s (1988) inviscid theory, a sufficient condition for a three-dimensional centrifugal-type instability of an axisymmetric flow is that the so-called Rayleigh discriminant, defined as

$$\Phi(r) = \frac{1}{r^3} \frac{\partial}{\partial r} \left( r^2 \nu_\theta^2 \right)$$

becomes negative somewhere in the flow, or, equivalently, that the local inviscid growth rate $\sigma$, given by $\sigma = (-\Phi)^{1/2}$, takes real positive values. Figure 30a shows the radial distribution of $\sigma$ in the present case.

Following Bayly (1988), an estimate of the global growth rate $S$ of the flow can be obtained by considering two additional effects attenuating the inviscid local growth. The first effect is linked to the finite radial extend of the unstable region. Perturbations of low wavenumber $k = 2\pi/\lambda$ (i.e., of large wavelength $\lambda$) are attenuated. The second effect is due to the influence of viscosity, which damps perturbations with high wavenumbers (i.e., small wavelengths). The main effect of viscosity is to subtract the term $c\nu k^2$ from the inviscid growth rate, where $\nu$ is the kinematic viscosity and $c$ a constant of $O(1)$.

![Figure 30](image-url)
Since the precise value of $c$ is not known, $c = 1$ is chosen to keep the analysis as general as possible.

Then the global viscous growth rate is given, in dimensional units, by:

$$s = \sigma_0 - \frac{s_1}{k} - c\nu k^2 = \sigma_0 - \frac{s_1\lambda}{2\pi} - \frac{4c\pi^2\nu}{\lambda^2}$$

where:

- $k$ is the perturbation wavenumber
- $\lambda = 2\pi/k$ is the perturbation wavelength
- $\sigma_0$ is the local inviscid growth rate defined as: $\sigma_0 = \sigma_{\text{max}} = \sigma(r_0)$. Figure 30(a) gives $\sigma_0 = 2.72$ s$^{-1}$.
- $s_1$ is given by: $s_1 = (-\sigma''\sigma_0)^{1/2}$

By non-dimensionalising $s$ as follows, the non-dimensional global viscous growth rate is obtained:

$$s^* = \frac{2\pi b_0^2 s}{\Gamma} = \frac{2\pi b_0^2 \sigma_0}{\Gamma} - \frac{s_1 b_0}{\Gamma} \left(\frac{\lambda}{b_0}\right) - c\frac{8\pi^3 \left(\frac{\lambda}{b_0}\right)^2}{\text{Re}_T} = 2.22 - 2.99 \left(\frac{\lambda}{b_0}\right) - c\frac{8\pi^3 \left(\frac{\lambda}{b_0}\right)^2}{\text{Re}_T}$$

where:

- $\sigma_0 = 2.72$ s$^{-1}$
- $\sigma'' = -1315$ s$^{-1}$ cm$^{-2}$
- $s_1 = (-\sigma''\sigma_0)^{1/2} = -59.8$ s$^{-1}$ cm$^{-2}$
- $\nu = 10^{-2}$ cm$^2$ s$^{-1}$
- $b_0 = 2.6$ cm
- $\Gamma = 52$ cm$^2$ s$^{-1}$

The global viscous growth rate, as well as the contribution of the two effects described above, is plotted figure 30b for the present case. This result tends to validate the hypothesis of a centrifugal-type instability since the maximal value of the non-dimensional growth rate is of order 1. Moreover, the most unstable wavelength given by this analysis ($\lambda = 0.26b_0$) is in very good agreement with the experimental observations (LIF visualizations give $\lambda = 0.25b_0$, see figure 27).

In conclusion, arguments have been given to support the hypothesis of a centrifugal-type instability. Experimental observations are in good agreement with existing centrifugal instability theory for two-dimensional flows with closed streamlines. Physical arguments may be added to this analytical work. The vortex pair is experimentally generated by two plates and its motion history is characterized by an exponential acceleration followed by a smooth deceleration before the end of the motion (see section 3.1 and figure 8). As mentioned earlier, the plate’s motion has been experimentally optimized to generate vortices without perturbations. As shown by figure 26, no instability develops for relatively low Reynolds numbers. At $\text{Re}_T = 5200$, however, the centrifugal-like instability is very well developed. Since higher Reynolds...
numbers imply higher angular velocity imposed on the plates, the deceleration and the final step in the velocity profile (from the last non-zero velocity to zero) are more important. As a consequence, the strength of the opposite-signed vorticity generated at the end of the motion is higher as the Reynolds numbers increases. Opposite-signed vortex sheets are likely to roll up into the primary vortices during their generation, leading to a region of decreasing circulation in the azimuthally averaged circulation profiles of the vortices. In that case, the conditions for the centrifugal instability (as verified analytically) are satisfied.

4.2.2 Second short-wavelength instability

Both experimental (figure 26, last row of images) and numerical investigations show that a short-wavelength instability develops. Numerical simulations show clearly how the instability first develops in the initially uniform secondary vortex and how the interactions between the primary and secondary vortices lead to the perturbation (instability of similar short-wavelength) of the primary one. Eventually, this mechanism leads to the decay of the whole system. In the present work, experimental investigations show that an instability subsequent to the centrifugal-type one described above, and of larger wavelength, develops. Extensive volume LIF visualizations have been performed in order to gain more knowledge on the development of this short wavelength instability into the secondary vortex, and on the dynamics of the latter. At $Re_T = 5200$, a configuration comparable to numerical simulations, the development of the centrifugal instability makes the flow very complex, hence the visualizations difficult.

Figure 31 shows a longitudinal cross-section, similar to that of figure 27, at two non-dimensional times. The development of the two successive instabilities appears clearly. At $t^* = 2$, the primary vortex reaches the minimum altitude before rebound (see the corresponding vorticity field in figure 31), the boundary layer starts detaching, but the vortex sheet has not wrapped around the primary one yet. The corresponding longitudinal cross-section LIF visualization shows that, at that time, the centrifugal-like instability is well developed. At $t^* = 5.2$, the primary vortex is out of the laser sheet, but light scattering allows visualizing it, showing a second instability of longer wavelength. The primary vortex core starts being perturbed. The vorticity field shows that, at that time, the secondary vortex has orbited around almost the whole primary one. The periodic structures visualized by LIF are likely to be: either the secondary vortex wrapped around the primary one (as observed by simulation in figure 24), or some deformations of the primary vortex core due to the phenomenon described in figure 28. In both cases, the structures are a consequence of the short-wavelength instability of the secondary vortex.

In order to visualize clearly the second vortex, a third kind of volume LIF visualizations has been carried out. The flow is still illuminated with a cone of laser light. Two different kinds of dye are used: Fluorescein (green color), as used in the visualizations
Figure 31: Same configuration as in figure 27, but at two successive non-dimensional times (left: $t^* = 2.0$, right: $t^* = 5.2$) with the corresponding vorticity field in the cross-section. Scattered laser light allows visualizing the primary vortex even outside laser sheet plane (for $t^* = 5.2$).

above, is injected on the ground to visualize the dynamics of the boundary layer and secondary vortex. Rhodamine (red color) is painted on the plate tips as described in 3.2 to visualize the primary vortices. The flow is recorded from above the ground by a digital camera positioned with an angle, in order to obtain three-dimensional pictures similar to those obtained by numerical simulations (see figure 25).

Figure 32 shows a series of visualizations obtained for $Re_f = 5200$. At $t^* = 0.6$, the Fluorescein (green dye) is lying on the ground and the background primary vortex (red dye) is approaching it. Since the primary vortices are strongly perturbed (due to the centrifugal-type instability) when they reach their minimum altitude, the vortex sheet detaching from the ground (visualized in the foreground at $t^* = 2.5$) is already perturbed. As the perturbed secondary vortex wraps around the primary one ($t^* = 3$), the flow becomes highly complex showing characteristic vortex structures wrapped around the primary vortex. As a result, the visualization of the second instability is very
Figure 32: Volume LIF visualizations for $Re_T = 5200$. Fluorescein (green dye) is injected on the ground to visualize the boundary layer and secondary vortex; Rhodamine (red dye) is painted on the flap tip to visualize the primary vortex (only the background one for visualization matters).

difficult ($t^* = 4$) and the bursting of the whole system into small scale turbulence ($t^* = 5.3$) is enhanced.
Figure 33: Same three-dimensional LIF visualizations as in Figure 31, $Re_T = 2500$. Only the background vortex (red dye) is visualized, Fluorescein (green dye) is injected on the ground to visualize the dynamics of the boundary layer and detaching vortex sheets.

In order to visualize the development of the second short-wavelength instability, it is necessary to avoid the first centrifugal-type instability. For that purpose, the same kind of three-dimensional LIF visualizations has been carried out at a lower Reynolds number. Figure 32 shows the series of visualizations obtained for $Re_T = 2500$. At $t^* = 1.8$, the Fluorescein (green dye) is lying on the ground and the background primary vortex is approaching the ground. At $t^* = 3.2$, wall vorticity has detached and rolled up into an almost unperturbed secondary vortex (structure visualized in the foreground). At $t^* = 4.6$, an instability has developed on the secondary vortex which is wrapping around the primary one (not visualized). At $t^* = 5.2$, the secondary vortex is wrapped around the primary one. At this low Reynolds number, the decay is much slower due to the intrinsic Reynolds number effect. The effect of the development (at $Re_T = 5200$) or not (at $Re_T = 2500$) of the centrifugal-type instability is still an open issue. Quantitative measurements and investigations at $Re_T = 5200$ in the absence of centrifugal-type
instability but for otherwise identical conditions would be needed. A first conjecture is that the small vortex structures resulting from the centrifugal-like instability (figure 27) have negligible circulation and no effect on the dynamics of the primary vortex. A second one is that the intense bending and stretching of the secondary vortex due to the centrifugal-type instability (figure 32, $t^* = 3$) enhance the decay of both the primary and secondary vortices.

Although the experimental set-up appears to produce undesirable effects at a Reynolds number of 5200, since it generates centrifugal-type instabilities, the short-wavelength instability has been experimentally observed at lower Reynolds number. According to the present experimental work and to the numerical simulations performed at UCL, the development of this instability is characteristic of the dynamics of a counter-rotating vortex pair in the vicinity of the ground in idealized conditions. The investigations at $Re = 5200$ in the presence of an external perturbation (centrifugal-type instability) tend to show that the dynamics may be greatly affected in non-idealized conditions (high turbulence…). But this is out of the scope of this work and the interested reader will find detailed studies of these effects in the literature (see section 2.3.2).

Concerning the dynamics in idealized conditions, the present work, as well as the simulations, is in agreement with previous investigations. In their three-dimensional investigation of the interactions of the primary and secondary vortices by numerical simulations, Luton & Ragab (1997) compared the short wavelength instability of the secondary vortex to Widnall (1974) instabilities. As stated in section 2.2, previous investigations showed that these instabilities are of the kind occurring in vortical flows with elliptical streamlines. In the present work, two-dimensional PIV measurements show that the secondary vortex has an elliptical ‘shape’. Using these measured flow fields (see figure 23), it is possible to estimate the wavelength and growth rate of the elliptic instability, based on theoretical consideration, and to compare these predictions with the experimental observations. This analysis will be carried out separately.
4.3 Crow instability development for large initial heights and interaction with the ground

In this section, results obtained for a large initial height are presented. The development of the Crow instability and the impact of the resulting periodic vortex rings are investigated. The base flows investigated are characterized by an initial height \( h_0 = 6b_0 \), an initial vortex separation distance \( b_0 \) of about 2.5 cm and two Reynolds numbers of about 3000 and 4000. The Crow instability is forced, as described in section 3.2, leading to the reconnection of the two vortices and to the interaction of a pair of periodic vortex rings with the ground. The configuration studied hereafter is of double interest. First, it enables the validation of the set-up and the base flow. To do so, flow parameters are compared to previous investigations of Crow instability OGE, since ground effect appears to be effective here only after reconnection. The methods used to analyze the data are described in detail in this section. Second, the interaction of periodic vortex rings resulting from reconnection is of fundamental interest. As discussed in section 2.3.2, the interaction of a single vortex ring has previously been studied in detail while, to the knowledge of the author, the interaction of periodic vortex rings resulting from Crow instability has never been the object of significant investigations.

As mentioned earlier, the results presented below are obtained with an initial sinusoidal perturbation imposed at the plate tips (wavelength of about \( 4.8b_0 \) and amplitude of about \( 0.04b_0 \)). It is important to note the dependence of the quantitative results on the initial perturbation. Initial parameters need to be defined rigorously.

Before analyzing the results obtained with this configuration, an experiment without any perturbation has been carried out (figure 34). The growth rate of Crow instability in the absence of imposed perturbations is relatively low and the vortices remain straight during a relatively long period of time (allowing the study of straight vortex pairs for low enough initial heights as presented in the previous sections). The wavelength of the most amplified mode is known to be dependent on initial perturbations; i.e. random noise in our case. This makes the control of our initial flow conditions very difficult. Thus, a sinusoidal perturbation of amplitude much higher than the random noise is imposed in order to control the flow and make the experiments repeatable (constant wavelength).
Figure 34: Bottom and side views of Crow instability development in the case of non-forced instability, $h_0 = 6.6b_0, \text{Re}_f = 3000$.

Figure 35 shows the logarithm of the non-dimensional Crow amplitude versus non-dimensionalized time, obtained with initial perturbations, and measured using the procedure described in section 3.2 (figures 9a and 10). The linear part of the curve,
observed after a noisy initial development of the instability, is in agreement with theoretical results predicting an exponential growth OGE. The non dimensional growth rate, \( \sigma^* = \sigma \left( \frac{2 \lambda b_0^2}{\Gamma_0} \right) \), is given by the slope of the curve, and the initial amplitude, \( A_0 \), is obtained by extrapolating the linear curve to \( t^* = 0 \). For the configuration studied in this section (\( Re_\Gamma = 3000 \) with initial perturbations), figure 35 gives: \( A_0 = 0.03b_0 \) and \( \sigma_0^* = 0.83 \), in excellent agreement with the theoretical growth rate, which is close to 0.8 in a range of \( r_c/b \) and \( \lambda/b \), including the present experimental values.

Figure 35: Crow instability amplitude obtained with forced perturbations (wavelength : 4.8\( b_0 \) and amplitude : 0.04\( b_0 \)) showing the exponential growth (the logarithm being linear) once the instability is well developed.

Figure 36 shows front (the vortex pair is moving towards the observer) and simultaneous side views at successive non-dimensional times in the case of forced perturbations. Reconnection occurs before the presence of the ground affects the dynamics. Periodic elliptical rings result from Crow instability development, which then interact with the ground. One switch of the rings’ major axes, as described by Leweke & Williamson (1998b) is observed. Then, similar to the observations made on a single ring, the periodic rings are stretched in a plane parallel to the ground due to inviscid effects (interaction with their images). The next step is the rebound of the rings, while stretching still goes on. For the case studied, the rebound is different from that of a single vortex ring: the two portions of a given ring facing adjacent vortex rings rebound, while the other two lateral parts do not (or very weakly).

Figure 37 shows the vortex centre positions obtained from visualizations. The peak plane corresponds to the cross-section of the two parts that do not rebound. The fact that the whole ring does not rebound implies that this rebound is, for the present set of parameters (\( Re_\Gamma, b_0, a_0 \), initial forced perturbation), not due to secondary opposite-
Figure 36: Front and side views for $h_0 = 6.0$ and $Re_\Gamma = 3000$. The Crow instability process is observed, as well as stretching and rebound of vortex rings.
signed vorticity created at the wall by viscous effects. Indeed, if this were the case, the whole ring would rebound, as observed for a single vortex ring. This is likely due to the fact that at this time, the strength of the thin remaining vortices observed in the peak plane on figure 36 ($t^* = 12.7$) is too small to generate significant secondary vorticity at the wall. In the present case, the opposite-signed vorticity causing the rebound of the corresponding part of the rings comes from the neighbouring vortex rings. At late times, the flow is characterized by periodic turbulent ‘columns’ of rising fluid, connected to each other by thin persisting vortices as observed at $t^* = 12.7$ and 18.8. In figure 37b, the origin is located at ground level at the $x$-position of the centre point between the two initial vortices (at $t^* = 0$). For figure 40, a run with a non-symmetric evolution was chosen intentionally, in order to illustrate the effect of symmetrisation of the trajectory. The symmetrised plot shows how the trajectory would be in ideal conditions. Slight asymmetries observed may easily be explained by the weak background fluid motion (mainly due to convection) that may remain in the water tank and the slight asymmetry in the alignment of the set-up, since the flow is extremely sensitive to these conditions. In the results presented hereafter, only minor asymmetries were observed in the trajectories, and their influence on the overall dynamics can be neglected.

![Figure 37](image)

At higher Reynolds number, similar features are observed. For a Reynolds number of about 4000 however, the short wavelength instability appears clearly during reconnection (figure 38). The result is that the breakdown of the vortex structure into
turbulence is greatly enhanced, as observed by Leweke & Williamson (1998a) for a vortex pair OGE at Reynolds numbers above 2000.

Figure 39 shows the evolution of the vortex spacing in the two planes of the Crow instability (peak and trough), obtained from cross-cut visualizations. Vortex spacing is non-dimensionalised by its value at the initial time $t_0$. Both dimensional and non-dimensional plots are presented. During the vortex generation, while the plates are moving, the vortex spacing decreases. Once the plates stop, the vortex pair moves towards the ground. While the vortex pair remains OGE, the trajectory of the vortices is perpendicular to the wall, and the induced velocity and vortex spacing are constant.

The initial time $t_0$ is defined as the time at which this OGE phase starts, after the vortex generation. It has been observed that $t_0$ corresponds to the end of the plate motion. Initial parameters $b_0$ and $V_0$ are obtained by time-averaging during this phase the vortex spacing and velocity displacement, respectively. As the vortex pair is approaching the ground, its dynamics start being affected. At this point, the Crow instability has already developed. Fogg (2001) showed that the peak and trough planes may have very different dynamics in Ground Effect (IGE) and that reconnection may not occur. For high initial heights, such as the one considered here, this is not the case. Reconnection occurs (the vortex spacing in the trough plane decreases to zero) before the vortices spread laterally away from each other due to the proximity of the ground, resulting in the series of periodic rings shown on figure 36. In the peak plane, the vortex spacing increases continuously due to ground effect.

The vertical position of the vortices in both planes is presented figure 40. For each plane, the scattering of the different curves (right, left and symmetrised vortices) is a good estimation of the uncertainties due to the non-ideal conditions. In the OGE region, the curves match very well. The slope obtained with the dimensionless vertical position and the dimensionless time is unity, as it should be by definition of $t^\ast$. Then (below $h_0 = 4b_0$), the rate of descent of the vortices in the peak plane slightly decreases due to the proximity of the ground. At the same time, the rate of descent of the vortices in the trough plane increases before reconnection, and a small ‘rebound’ is observed.
During take-off and landing, the altitude of the aircrafts flying over the runway is too low for reconnection to occur. Thus, the work described above concerning the rebound of periodic vortex rings is not directly related to the aeronautical application. However, the physical mechanisms comparable to those occurring in the case of a single vortex ring are of fundamental interest.
4.4 Summary and discussion

In the present investigation, the dynamics of longitudinally uniform vortex pairs near the ground at moderate Reynolds numbers have been studied experimentally in a water tank. Two initial heights of the pair were considered: a small height (two initial vortex spacings) and a large height (six initial spacings). In the framework of the project FAR-wake which aims to gain advanced fundamental knowledge on the dynamics of aircraft wake vortices generated during take-off and landing and interacting with the ground, the more relevant part of this work concerns the results obtained for low initial height. Both two-dimensional and three-dimensional dynamics have been investigated for this case, and are in good overall agreement with previous investigations. Vorticity is generated at the wall as the primary vortices approach the ground. Vortex sheets detach from the ground and roll up into secondary vortices as they wrap around the primary ones. These newly formed vortices cause the primary ones to rebound and to have loop trajectories. As the Reynolds number increases, the loops formed by the vortex trajectories are larger and the bursting into small-scale turbulence is enhanced. During the successive rebounds, the primary vortices remain at a distance from the ground between $0.6b_0$ and $1.3b_0$. Measurements of the circulation in two-dimensional cross-sections show that, as the vortex approaches the ground, the total circulation (of a half-plane containing one primary vortex only) decreases drastically up until the first rebound. However the primary vortex decays slowly, implying that the opposite-signed vorticity increases significantly (as observed) to cause the total circulation to decrease.

Once the primary vortices reach the maximum rebound altitude, some signs of three-dimensionality have been observed. The three-dimensional investigation shows that a short-wavelength instability develops first in the secondary vortex, affecting subsequently the primary one as the secondary perturbed vortex wraps around it. This mechanism enhances greatly the bursting of the whole system into small-scale structures. Experimental results are in good agreement with numerical simulations. However, a new instability is observed experimentally at high Reynolds number (>5000). It is shown by comparison with existing theoretical work that it is of the centrifugal type, and its origin is attributed to the vortex generation device. Although the development of this centrifugal-type instability is of interest from a fundamental point of view, the present work shows that this kind of experimental apparatus has to be used with care at high Reynolds number when studying vortices in ideal conditions. Some additional optimizations of the plate motion may help avoiding the development of such instabilities.

Typical Reynolds numbers encountered in the wake of civil aircrafts are of the order of $10^3$, while the order of magnitude of the Reynolds number obtained in experimental studies ranges approximately from $10^1$ to $10^4$. This difference between the experiments and the real flows, as well as its effects on the relevance of the results, is a challenging question in the field, and only some general points may be discussed here. As the Reynolds number increases, the viscous time, hence viscous dissipation, becomes negligible compared to the convective one. As a consequence, vortices generated at high Reynolds numbers are expected to persist much longer than at low ones. No certitude
really exists, but observations and measurements carried out during real flight tests show similar dynamics to that observed in the experiments. Another point of divergence to take into account is the parameter $a/b_0$ which is much lower for aircraft wakes than in water tank experiments. This parameter is likely to affect the development of the three-dimensional instabilities. The question regarding its precise influence could be addressed with the help of results from numerical simulations, such as those performed at UCL, where the parameter $a/b_0$ is closer to real conditions than the one in the experiments presented here.

Nevertheless, recalling the main objective of the task 3.1 of FAR-Wake, which is to gain new fundamental knowledge about ground interactions in idealized conditions, the present results and their good agreement with numerical simulations are of great interest. Some extensions of the present work would be also interesting. The stability analysis of two-dimensional vorticity fields to predict the development of short-wavelength elliptic instabilities would validate the hypothesis of an elliptical instability. Such studies are planned at CNRS-IRPHE. Comparison of the wavelengths between experimental, numerical and analytical results will show if the results from these three different approaches agree. Additional LIF visualizations and PIV measurements at $Re_l = 5000$ in the absence of centrifugal-type instability would improve the comparisons mentioned above. Three-dimensional LIF visualizations using two different dyes, a very difficult procedure, could be improved to obtain additional explicit visualisations of the three-dimensional dynamics.

Although not directly related to the main FAR-Wake objectives, it is worth mentioning the qualitative results concerning the interaction of periodic vortex rings with the ground obtained during the experiments carried out at large vortex generation height. This body of work is less relevant for the application to the interaction of aircraft wake vortices with the ground encountered in airports, since typical aircraft altitudes are too low to lead to reconnection and formation of periodic rings before ground interaction. However, this topic is interesting from a fundamental point of view. The main difference with the extensively studied case of a single vortex ring impacting on a wall is that, at the Reynolds number considered, the rings do not rebound as a whole. Only the parts of the rings influenced by the counter-rotating vorticity of the adjacent rings do rebound. This leads to a characteristic situation with periodic ‘columns’ of rising turbulent fluid, linked by two thin vortices lying close to the ground and for which circulations are too weak to generate secondary vorticity which would induce upward motion. Enhancement of vortex breakdown during ground interactions, due to the development of an elliptic short-wavelength instability similar to that observed OGE, has also been observed. Surprisingly, such a flow does not seem to have been the object of significant investigations in the past.
5 Conclusion

The dynamics of temporally evolving, longitudinally uniform, counter-rotating vortex pairs have been investigated experimentally in a water tank. Laser Induced Visualizations as well as Particle Image Velocimetry measurements have been carried out.

The work presented here includes two parts. The experimental apparatus and flow conditions have been validated at a large initial vortex generation height (six times the initial vortex separation distance). In addition, these experiments produced fundamental results about the interaction of periodic rings, resulting from late development of Crow instability, with the ground. The main result is that at the Reynolds numbers considered, and contrary to the case of a single vortex ring impacting a wall, the secondary vorticity seems to be too weak to induce the rebound of the whole rings. Only the parts adjacent to neighbouring vortex rings lift up, due to the velocity induced by the latter. A characteristic flow structure involving periodic columns of rising turbulent fluid is then observed at late times.

The main part of this work concerns the dynamics of a pair of uniform vortices, generated at an altitude of two vortex spacings, which is low enough for the development of the Crow instability to be negligible before the interaction with the ground takes place. The two-dimensional investigation showed the generation of secondary vorticity at the ground, which lifts off and rolls up into secondary vortices of opposite sign than the primary ones, inducing in turn a rebound of the latter. Visualisations and velocity measurements allowed a characterisation of the trajectories and the evolution of circulation. The results are in agreement with previous studies. They show some characteristic features three-dimensionality in the flow. These three-dimensional interactions between the primary and secondary vortices are characterized by the development of a short-wavelength instability in the secondary vortex, subsequently affecting the primary one. A good overall agreement has been obtained with the numerical simulations performed at UCL. Analysis of the data obtained during the present work are likely to provide additional knowledge on the origin of the short-wavelength instability, confirming the hypothesis of an elliptical instability of the secondary vortex.. A new centrifugal-type instability has been observed at high Reynolds number and before ground interactions. Experimental and analytical investigations show that it is generated by the vortex generation system. The development and effects of this instability on the later evolution of the flow are discussed in detail.
REFERENCES


KORNEV, N.V. & REICHERT, G., 1997, Three-dimensional instability of a pair of trailing vortices near the ground, AIAA J. 35(10), 1667-1669.


PROCTOR, F.H. & SWITZER, G.F., 2000, Numerical simulation of aircraft trailing vortices, In “9th Conference on Aviation, Range and Aerospace Meteorology”, 511-516, Orlando (FL), USA.


TOMBACH, I., CROW, S., & BATE, E., 1975, Investigation of vortex wake stability near the ground, Report AV-FR-538, AeroVironment Inc., Pasadena (CA), USA.


TÜRK, L., COORS, D. & JACOB, D., 1999, Behaviour of wake vortices near the ground over a large range of Reynolds numbers, Aerosp. Sci. Technol. 3(2), 71-81.

