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**FAR-Wake**

Fundamental Research on Aircraft Wake Phenomena

Specific Targeted Research Project

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Duration: 36 months

LES calculations of spatially evolving wakes in ground effect

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Abstract

The case of a longitudinally developing wake generated by an elliptic wing is investigated at a moderate yet significant Reynolds number ($\Gamma_0^* / \nu = 10,000$), and using LES. The exact same vortex wake is also investigated in a longitudinally uniform configuration (i.e., a temporal simulation with periodic boundary conditions) using the same code and LES subgrid scale model. Finally, the 2-D (thus necessarily time developing) DNS of the same case is also performed, as a reference. For the two LES, the simulations are performed up to fully developed turbulence.

For each configuration, the averaged vorticity field, vorticity fluctuation field, and axial velocity field are presented and discussed. For the 3-D configurations, the flow topology evolution is also analyzed. Finally, the time evolution of diagnostics characterizing the wake dynamics are presented: trajectories, circulation, energy, etc.

The results show that, in the initial phase, the flow is essentially two-dimensional. During that phase, the 3-D results match very well with the 2-D reference configuration. One important feature, that is not present in the longitudinally uniform configuration, is the fact that the longitudinally developing wake induces an axial velocity defect in the primary vortices. The longitudinally developing wake therefore exhibits an earlier and more violent transition to turbulence than the longitudinally uniform configuration. The longitudinally developing configuration further differs from the uniform case by the fact that the vortex tubes describe a spiralling motion (i.e., they are not parallel). This feature is suspected to trigger some kind of vortex meandering, only observed in the longitudinally developing configuration.
## Contents

1 Introduction ........................................... 4

2 Definition of the longitudinally developing wake ............. 4

3 Numerical codes ...................................... 7
   3.1 Two-dimensional vortex particle method .................. 7
   3.2 Three-dimensional combination of Vortex-in-Cell and Parallel Fast Multi-pole methods .................. 8

4 Simulation configurations ................................ 10
   4.1 Two-dimensional setup .................................. 10
   4.2 Three-dimensional time developing setup ................ 10
   4.3 Three-dimensional space developing setup ............... 11

5 Results .................................................. 12
   5.1 Global wake dynamics ................................... 12
   5.2 RMS vorticity field fluctuations ......................... 13
   5.3 Axial flow component .................................. 14
   5.4 Three-dimensional wake topology ......................... 15
   5.5 Time evolution of diagnostics ........................... 15

6 Conclusion ............................................. 17

References .............................................. 43
1 Introduction

The objective of this contribution to the FAR-Wake project is to investigate the effects related to the longitudinally developing character of an aircraft wake generated in ground effect. For practical reasons (mainly computational cost and numerical complexity), one usually makes the hypothesis of a longitudinally uniform wake, thus allowing for simple periodic boundary conditions in the longitudinal direction: starting from a given (longitudinally uniform) initial vortex wake, its temporal development is investigated. In reality, however, the vortex wake generated by a wing is not longitudinally uniform: the vortex sheet rolls up in a spiralling manner, leading to non parallel vortex lines. Furthermore longitudinally non-uniform effects arise from the presence of the wing at the origin of the vortex wake: the vortex lines are not infinite, but closed at one end, by the wing’s circulation.

In order to investigate these effects, the case of a longitudinally developing wake generated by an elliptic wing is here investigated at a moderate Reynolds number and using LES. The numerical code is a combination of the Vortex-In-Cell method and the Parallel Fast Multipole method (VIC-PFM). The exact same vortex wake is investigated in a longitudinally uniform configuration (i.e., a temporal simulation with periodic boundary conditions) using the same code and LES subgrid scale model. Finally, the 2-D (thus necessarily time developing) DNS of the same case is also performed as a reference.

This document is organized as follows. In Section 2, we define the parameters characterizing a spatially developing wake. The 2-D and 3-D numerical codes are briefly presented in Section 3. The simulation parameters for the three cases (two-dimensional, longitudinally uniform and longitudinally developing) that were investigated are then presented in Section 4. Finally, in Section 5, we present and compare the simulations results.

2 Definition of the longitudinally developing wake

We consider the wake generated by an elliptic wing of wing span $b$. For a given aspect ratio $A_r$ and wing surface $S$, it is characterized as follows:

$$
\begin{align*}
    b &= \sqrt{S \times A_r} \\
    \bar{c} &= \frac{\sqrt{S}}{A_r} \\
    c(y) &= \frac{4}{\pi} \bar{c} \sqrt{1 - \left(\frac{y}{b/2}\right)^2}
\end{align*}
$$

where, $b$ is the wing span, $\bar{c}$ is the mean chord and $c(y)$ is the chord distribution.

We here use $A_r = 7.5$. This value is typical of a commercial aircraft. Is is also similar to the wing configurations used in the experimental campaigns by UCL and DLR in FAR-Wake Task 3.1.2. The span loading of the elliptic wing is evaluated taking the ground effect into account using a modified “lifting line” theory, as was done in FAR-Wake deliverable D3.1.1-1 (Span loading variations and wake roll-up in ground effect [4]). It should be noted that, unless otherwise mentioned, we define the reference quantities (circulation, vortex...
spacing, characteristic time, etc) based on those of the same elliptic wing out of ground effect; as was also done in [4]. Those quantities are written using a star as superscript.

The configuration is further defined by the height of the wing above the ground, \( h/b \), and by the wing lift coefficient, \( C_L = L/(\frac{1}{2} \rho U_\infty^2 S) \). The choice of these parameters are essential, as they have a direct effect on the spatial development of the wake but also on the computational requirements of the longitudinally developing simulation.

As was shown in [4], the global dynamics of a vortex sheet roll-up IGE are very similar for different initial heights above the ground: the vortex sheet itself rolls up very rapidly forming the primary vortex and, on the ground, the boundary layer (BL) separates and constitutes a secondary vortex (of opposite strength) that orbits around the primary vortex. The essential trend when the height above the ground decreases is that the dynamics are globally accelerated. More precisely, the time needed for the secondary vortex to separate and to perform one complete orbit around the primary vortex is almost directly proportional to the initial height. Thus, in a longitudinally developing configuration, the distance to perform one orbit is directly related to the initial height: the higher the location of the wing is above the ground, the longer the longitudinal distance required to perform one orbit.

We will now establish the relation between the lift coefficient \( C_L \) and the vortex wake characteristic time, \( T_0 = b_0/V_0 \) (with \( V_0 = \frac{\Gamma_0}{2\pi b_0} \), and where \( \Gamma_0 \) is the total circulation of the half vortex wake and \( b_0 \) is the spacing between centroids of vorticity). We can express the lift in terms of the circulation along the wing:

\[
L = \rho U_\infty \int_{-b/2}^{b/2} \Gamma(y) dy, \tag{1}
\]

The wing circulation, \( \Gamma(y) \), is related to the wake vortex sheet by \( \gamma(y) = -\frac{d\Gamma}{dy}(y) \). It can thus be related to the wake vorticity characteristics \( \Gamma_0 \) and \( b_0 \):

\[
\int_{-b/2}^{b/2} \Gamma(y) dy = -\int_{-b/2}^{b/2} y \frac{d\Gamma}{dy} dy = \Gamma_0 b_0. \tag{2}
\]

Finally, we obtain the lift coefficient as:

\[
C_L = 2 \frac{\Gamma_0 b_0}{U_\infty S}. \tag{3}
\]

This relation can be used to express \( T_0 \) as a function of the lift coefficient:

\[
T_0 = \frac{4\pi b_0^3}{C_L U_\infty S} \simeq \left( \frac{\pi}{2} \right)^4 \frac{A_r}{C_L} \frac{b}{U_\infty}. \tag{4}
\]

The approximate equality is obtained by assuming that \( b_0 \simeq \frac{\pi}{2} b \) (which is exactly true for an elliptical wing out of ground effect) and by remembering that \( A_r = b^2/S \). If we now define \( d_0 = U_\infty T_0 \), the characteristic distance travelled by the wing for the vortex wake to descend by one vortex spacing \( b_0 \), it appears that the wake development distance is directly related to the lift coefficient:

\[
d_0 = \frac{4\pi b_0^3}{C_L S} \simeq \left( \frac{\pi}{2} \right)^4 \frac{A_r}{C_L} b \tag{5}
\]
Table 1: Characteristic distance, $d_{rot}$, for the secondary vortex to perform one full orbit around the primary vortex, for two lift coefficients, and four initial heights.

<table>
<thead>
<tr>
<th>$h/b$</th>
<th>$t_{rot}/T_0$</th>
<th>$d_{rot}/b$</th>
<th>$CL = 1.5$</th>
<th>$d_{rot}/b$</th>
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<tr>
<td>0.125</td>
<td>0.6</td>
<td>17.1</td>
<td>4.26</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.25</td>
<td>1.4</td>
<td>39.8</td>
<td>9.94</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td>2.9</td>
<td>82.4</td>
<td>20.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.0</td>
<td>6.0</td>
<td>170</td>
<td>42.6</td>
<td></td>
<td></td>
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Table 2: Span loading characteristics for an elliptic wing IGE ($A_r = 7.5$). The span loading is characterized by $\Gamma_0$ and $b_0$ and is compared to $\Gamma_0^*$ and $b_0^*$ for the reference configuration: the elliptic wing (same aspect ratio and span) OGE.

<table>
<thead>
<tr>
<th>$h$</th>
<th>$\Gamma_0/\Gamma_0^*$</th>
<th>$b_0/b_0^*$</th>
<th>$\Gamma_0/b_0^*$</th>
<th>$\Gamma_0^<em>/b_0^</em>$</th>
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<tr>
<td>$\infty$</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>1.0000</td>
<td>1.0062</td>
<td>1.0010</td>
<td>1.0219</td>
<td>1.0610</td>
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<tr>
<td>0.7500</td>
<td>0.9998</td>
<td>0.9995</td>
<td>0.9982</td>
<td>0.9911</td>
</tr>
<tr>
<td>0.5000</td>
<td>0.9996</td>
<td>0.9996</td>
<td>0.9983</td>
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<td>1.0000</td>
<td>1.0000</td>
<td>1.0001</td>
<td>1.0001</td>
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Based on this relation and on the results obtained in [4] for the roll-up of vortex sheets at different initial heights above the ground, Table 1 presents the characteristic distance for the secondary vortex to perform one full orbit around the primary vortex. Results are presented for four initial heights and for two lift coefficients. From these values, it clearly appears that, if one wants to capture a significant part of the wake dynamics (i.e., at least one full orbit), the domain length in the longitudinal direction has to be very large. In order to limit the domain size one can choose a low initial height above the ground and an artificially high lift coefficient.

Following these considerations, we now define the case that is investigated in the longitudinally developing configuration. As already mentioned and as done in [4], the wing configuration is defined with respect to the configuration out of ground effect (OGE). Using the methodology developed in [4], the span loading of a wing OGE can be obtained for the same same wing configuration IGE: due to the ground, the wake characteristics are modified. Table 2 presents the span loading characteristics for different heights above the ground with respect to the reference configuration OGE. We chose to investigate the following case:

\[
A_r = 7.5 \\
C_L = 6.0 \\
h/b = 0.25 \\
L_x/b = 12.5
\]

The lift coefficient, $C_L$, is defined as the actual lift coefficient of the wing IGE. Thus,
using Eq. (3) and remembering that $A_r = b^2/S$,

$$C_L = 2 A_r \frac{\Gamma_0 b_0}{\Gamma_0^* b_0^*}$$  \hspace{1cm} (6)

remembering also that for the elliptic wing OGE, $b_0^*/b = \pi/4$, we can evaluate the wing velocity, $U_\infty$, for this particular configuration:

$$U_\infty = \left(\frac{\pi}{2}\right)^4 \frac{A_r}{C_L} \frac{\Gamma_0 b_0}{\Gamma_0^* b_0^*} \frac{b}{T_0^*} = 8.00 \frac{b}{T_0^*}$$ \hspace{1cm} (7)

The extent of the computational domain, $L_x/b = 12.5$, for the longitudinally developing configuration was chosen to be somewhat larger than $d_{rot} \simeq 10$ corresponding to this case. We can already mention that the longitudinally developing simulation of this case requires about 55 million grid points. A more realistic case, with $C_L = 1.5$ for example, would require a four times longer domain (if one wants to reach the same characteristic time in the wake) and thus a four times larger computational grid!

The Reynolds number of the flow, $Re = \Gamma_0^*/\nu$, was here set to 10,000 as it is the case that was most thoroughly investigated in [4] and is sufficiently high to exhibit a turbulent behavior.

The initial vortex sheet produced by the elliptical wing IGE is discretized as follows. Its circulation per unit length is $\gamma(y) = -\frac{\Gamma_0^*}{\pi b_0} y$ with $\Gamma(y)$ the span loading as provided by the computations in [4]. This singular vortex sheet is first regularized in order to produce a regular vorticity field. For this, we use convolution with a gaussian:

$$\omega_\sigma(y, z) = \frac{1}{2\pi\sigma^2} \int_{-b/2}^{b/2} \exp \left( -\frac{(y - y')^2 - (z - h)^2}{2\sigma^2} \right) \gamma(y') dy'$$ \hspace{1cm} (8)

where $b$ is the wing span, $h$ is the distance to the ground and $\sigma$ is the regularization parameter. We here used $\sigma/b = 2 \times 10^{-2}$. This is “thicker” than the regularization used in the vortex sheet roll-up simulations in [4], but we are here limited by the minimum vortex sheet thickness that we are able to capture in the 3-D simulations where the grid resolution is limited. The regularized vorticity field is then discretized using particles put on multiple layers across the sheet.

3 Numerical codes

Vortex methods are lagrangian methods and are therefore efficient to simulate incompressible unsteady flows (we refer the interested reader to the reviews in [3, 5, 6]): they have negligible dispersion error and good energy conservation. In 2-D we use a “purely” Lagrangian approach, while, in 3-D, we use a hybrid approach, known as “Vortex-in-Cell”.

3.1 Two-dimensional vortex particle method

The vortex particle method solves the vorticity formulation of the incompressible Navier-Stokes equations. In 2-D, the evolution of the vorticity is given by:

$$\frac{D\omega}{Dt} = \nu \nabla^2 \omega$$ \hspace{1cm} (9)
where $\omega_z = (\nabla \times \mathbf{u})$. The vorticity field is discretized using a set of $n$ vortex particles characterized by an area, $S_p$, and a circulation, $\Gamma_p = \int_{S_p} \omega \, dS$. The method then follows the position of the particles and the evolution of their circulation as follows:

$$\frac{dx_p}{dt} = \mathbf{u}_p, \quad \frac{d\Gamma_p}{dt} = (\nu \nabla^2 \omega)_p S_p.$$  \hspace{1cm} (10)

The induced velocity field is determined using a regularized Biot-Savart integral (we here use Gaussian regularization functions). The velocity is computed using a fast-multipole method which yields an $O(n \log n)$ computational cost instead of $O(n^2)$ if the Biot-Savart interactions were computed directly. The viscous diffusion is handled using a particle strength exchange scheme (PSE): the PSE requires an interaction of each particle with their respective neighboring particles (these are easily determined “on the fly”, together with the fast multipole velocity evaluation).

In order to be accurate, Lagrangian methods require a good overlap of the particle regularization function. Even if the particles are initially uniformly distributed, in time the particles are convected by the flow and the required particle overlap deteriorates. Therefore every few time steps, the old particle set is replaced by a new set located on a regular lattice: the particles are “redistributed”. This is done using high order interpolation schemes.

For unbounded flow simulations, the vortex particle method can be applied “as-is”. In order to simulate the flow above a viscous ground, one obviously needs to enforce a zero velocity at the wall. By symmetrizing the vorticity field with respect to the ground (as in the previous sections), we are able to enforce exactly a no-through flow over the whole (i.e. infinite) ground plane. This however does not enforce the no-slip condition at the wall, as required. The no-slip is enforced by emitting a vorticity flux at the wall:

$$\nu \frac{\partial \omega}{\partial n} = \frac{\triangle \gamma}{\triangle t}, \quad \triangle \gamma = u_{\text{slip}},$$  \hspace{1cm} (11)

where $u_{\text{slip}}$ is the slip velocity resulting from the symmetrized vorticity field. This flux must be emitted during a time $\triangle t$. In effect, the vortex sheet $\triangle \gamma$ is distributed to neighboring vortex particles. The no-slip boundary condition is only applied to a limited region where the viscous ground interaction actually occurs.

### 3.2 Three-dimensional combination of Vortex-in-Cell and Parallel Fast Multipole methods

The three-dimensional code (called VIC-PFM for short) is based on a different approach that combines Lagrangian and finite difference methods but also solves the vorticity form of the incompressible Navier-Stokes equations:

$$\frac{D \omega}{Dt} = \nabla \cdot (\mathbf{u} \omega) + \nabla^2 \omega + \nabla \cdot (\nu_{\text{sgs}} (\nabla \omega^s + (\nabla \omega^s)^T)), \hspace{1cm} (12)$$

where $\nu_{\text{sgs}}$ is the effective subgrid-scale viscosity and $\omega^s$ the “small-scale” part of the LES field. The numerical solution of equation (12) is sought in a two-fold manner. The convective part is evaluated using a Lagrangian approach, thus with negligible dispersion
Figure 1: Vortex particle method IGE. This figure shows the symmetrized vorticity field (in order to enforce no-through flow on the ground) and the extend of the viscous ground (where no-slip is enforced by imposing a vorticity flux at the wall). The zoom shows the vortex particle locations.

Errors. The time variation of the vortex particle strength, that includes both the vortex stretching and the dissipation terms, is solved on a regular grid, using 2nd order finite differences. Interpolation from the Lagrangian particles to the Eulerian grid is done using the $M'_4$ scheme [3, 6]. Once on the Eulerian grid, the stream-vector $\psi$ is evaluated by solving the Poisson equation

$$ \nabla^2 \psi = -\omega . \tag{13} $$

The velocity field, needed for convection and stretching, is then obtained by evaluating $u = \nabla \times \psi$ using again 2nd order finite differences. The global time marching procedure is carried out using a 2nd order leap Frog scheme for the convection and the Adams-Bashford scheme for the diffusion. Finally, the intrinsic divergence-free character of the vorticity vector field is imposed by a proper reprojection of the discrete vorticity field (which also requires solving a Poisson equation).

Particular to the present implementation of this procedure is the treatment of the boundary conditions on the grid when solving equation (13). At a given time step, the boundary conditions for $\psi$ are determined using a Green’s function approach via a Parallel Fast Multipole method (PFM). With this approach, the unbounded domain condition can be accurately approximated on a relatively small grid: only as large as the vorticity field itself. This allows for a significant reduction of the computational cost for solving equation (13) when compared with more classical methodologies [5, 2, 6]. Furthermore, the method can be parallelized using a domain decomposition method: the PFM code, which has a global view of the whole field, is used to obtain proper boundary conditions on each subdomain [2, 5, 6].
In order to enforce the no-through flow on the ground, the symmetrizing procedure (as described in the previous section) can be trivially extended to the 3-D case: the boundary conditions on the grid (for solving the Poisson equation) are obtained using the Green’s function approach on a properly symmetrized vorticity field. The no-slip condition at the wall is enforced through a vorticity flux at the wall, as in 2-D.

The LES modeling is done using a multiscale subgrid-scale model: the model acts on the small scale part of LES field, \( \omega^s \), only. It is obtained from the complete LES field, \( \omega \), using a compact discrete filter (applied iteratively). We here use the Regularized Variational Multiscale (RVM) model [1]. The advantage of such model is that it preserves the inertial range while providing dissipation at the high wave numbers: it is only active during the complex phases of the flow, while remaining inactive during the initial phase, when the vortices are essentially laminar.

4 Simulation configurations

We now discuss the particular setup and computational details for the three simulations that were performed. From now on, we refer to the longitudinally developing case as the “Space Developing” case (SD) and to the longitudinally uniform case as the “Time Developing” case (TD). The two-dimensional case is noted (2D).

4.1 Two-dimensional setup

For the two-dimensional simulation, the Lagrangian Vortex Element Method is used, as in [4]. The spatial discretization is here sufficiently fine to perform a well resolved simulation, i.e. a DNS. The 2-D simulation can thus be used as a reference for the initial stage of the flow which is essentially two-dimensional (at least for the time developing case).

The size of the redistribution lattice, \( \Delta \), is set to \( \Delta/b = 2.5 \times 10^{-3} \) leading to an initial number of about 6,000 particles. The time step for the simulation was set to \( \Delta t/T_0^* = 5 \times 10^{-4} \) (where \( T_0^* = b_0^*/V_0^* \), with \( V_0^* = \Gamma_0^*/(2\pi b_0^*) \)). The particles were redistributed every 4 time steps. By the end of the simulation, the number of particles was several hundreds of thousands.

4.2 Three-dimensional time developing setup

For the time developing configuration, periodic boundary conditions are used in the longitudinal direction. The domain length is \( L_x/b = 4 \). In the span-wise and wall-normal directions, open domain conditions are used. The initial condition is the same as the two-dimensional setup, extruded in the longitudinal direction. In order to trigger the development of three-dimensional instabilities, the initial condition was perturbed: a random displacement was imposed on the discrete particle locations. The magnitude, \( \epsilon \), of the random displacement was set to \( \epsilon/b = 10^{-4} \).

The resolution of the Vortex-in-Cell grid is set to \( \Delta/b = 10^{-2} \) and is uniform in all directions. The initial number of particle is about one million. The time step for the
simulation was set to $\Delta t/T_0^* = 5 \times 10^{-4}$. The particles were redistributed every 4 time steps. The flow was computed up to $t/T_0^* = 2.0$ (4000 time steps). By the end of the simulation, the number of particles was $8 \times 10^6$ and the size of the Vortex-in-Cell grid was $L_x/b = 4$, $L_y/b = 4$ and $L_z/b = 1.25$ (thus $N_x = 400$, $N_y = 400$ and $N_z = 125$ for a total of 20 million grid points).

### 4.3 Three-dimensional space developing setup

The setup of the space developing configuration is less “conventional”. It essentially consists of a lifting line (modeling the elliptical wing) shedding a vortex sheet. The wake vortex sheet then dynamically evolves in space and time, due to its own induced velocity and the velocity induced by the lifting line itself. In order to “close” the problem, an outflow boundary condition is imposed: this is done by enforcing symmetries on the flow. It leads to the following constraints on the vorticity field (here written for a $y$-$z$ outflow plane located at $x = 0$):

\[
\omega_n(-x, y, z) = \omega_n(x, y, z), \quad (14)
\]
\[
\omega_t(-x, y, z) = -\omega_t(x, y, z), \quad (15)
\]

where $\omega_n$ and $\omega_t$ are respectively the normal and the tangential components of the vorticity field. This ensures that the normal component of vorticity is conserved through the outflow plane and thus allows for a natural outflow condition for wake vortices.

Figure 2 shows a schematic of the spatially developing domain. The effective computational domain for the Vortex-in-Cell method extends between the inflow and outflow plane ($L_x/b = 12.5$). In the transverse directions the grid size is adapted automatically to the region where the vorticity is non zero. When computing the boundary conditions on the VIC grid however, the lifting line vorticity, the inflow vortex sheet and the complete symmetrized vorticity fields (ground and outflow symmetries) are taken into account. The resulting velocity field thus corresponds to a true open domain calculation of a lifting line and its wake above the ground.

The free flow velocity, $U_\infty$, was set such as to obtain the lift coefficient, $C_L = 6$: $U_\infty T_0^*/b = 8$. The same resolution as the time developing calculation is used: $\Delta/b = 10^{-2}$ and it is also uniform in all directions. The time step for the simulation was set to $\Delta t/T_0^* = 5 \times 10^{-4}$. The particles were redistributed every 4 time steps. The number of particles was about $14 \times 10^6$ and the size of the Vortex-in-Cell grid was $L_x/b = 12.5$, $L_y/b = 4$ and $L_z/b = 1$ (thus $N_x = 1250$, $N_y = 400$ and $N_z = 100$ for a total of 50 million grid points).

Compared to the time developing simulations (both 2-D and 3-D), the computation has here to be carried on for sufficiently long times for the flow to reach its regime state (i.e., its statistical equilibrium). Indeed, the flow is initialized with a spatially uniform wake. A complete flow through time, $t = L_x/U_\infty$, (i.e., the time for the flow to completely traverse the computational domain) is required to “evacuate” this initial condition. A visualization of the temporal development of the space developing simulation showed that the flow had reached a well established regime at $t/T_0^* = 2.5$ (thus after about 1.6 flow through times). Once this regime state was attained, the flow had to be computed further,
to attain statistical convergence of the time averaged fields of interest. The averaged fields were considered sufficiently converged at $t/T_0^* = 5$ after initialization (i.e., statistics were gathered over $2.5 \ T_0^*$). The complete simulation thus required 10,000 time steps.

5 Results

We now discuss the results obtained for the 2-D simulation (2D), the 3-D time developing simulation (TD) and the 3-D space developing simulation (SD). Instead of describing each case separately, we compare and analyze the differences between the three simulations from different point of views.

5.1 Global wake dynamics

The global wake dynamics for the three simulations are presented from Figures 3 to 5: vorticity contours are shown at distinctive times, $T^* = t/T_0^*$, in the right half plane. The 2D results correspond to “snap shots” of the two-dimensional vorticity field. For the TD simulations, the longitudinal component of the space averaged vorticity field is shown. In
the SD simulation, the vorticity field corresponds to the longitudinal component of the time averaged vorticity field in distinctive planes at fixed distances behind the lifting line. The location of the averaging planes are shown in Fig. 10. In this case, \( T^* \) is defined based on the distance in the wake: \( T^* = (x/U_\infty)/T_0^* \).

As can be seen from those results, the dynamics are essentially identical for all three simulations up to \( T^* = 0.75 \). A closer look at \( T^* = 0.25 \) shows that the separation occurs slightly earlier (in space and time) in the TD and SD simulations when compared to the 2D case. It should be noted that both 3-D results have a coarser grid than the 2D case: the boundary layer is thus not as well captured. This could explain the small observed difference between the 2D and TD results. The separation in the SD case occurs even slightly earlier and is related to the fact that lifting line induces an additional velocity gradient on the boundary layer, and thus enhances the boundary layer separation.

The TD case remains very close to the 2D results up to \( T^* = 1.25 \), indicating that the flow is essentially two-dimensional up to that time. After that time, when the secondary vortex interacts with the ground, the TD flow becomes turbulent. At \( T = 2.0 \), the primary wake vortex is strongly diffused and the secondary vorticity from the boundary layer has lost much of its coherence.

The SD case transitions much earlier to turbulence: at \( T^* = 1.0 \), it is clear that some 3-D instabilities are already present in the wake as the averaged vorticity field appears to be much more diffused than the 2D and TD cases. At \( T^* = 1.5 \), the maximum vorticity in the SD case does not exceed \( \Omega = \omega T_0^* = 50 \) while the TD case still has values above \( \Omega = 200 \). Note that the \( T^* = 2.0 \) plane is not available as the SD domain extends up to \( T^* = 1.56 \).

### 5.2 RMS vorticity field fluctuations

In order to further illustrate the onset of instabilities and the transition to turbulence in both 3-D cases, Figures 6 and 7 present RMS vorticity fluctuations contours (longitudinal and tranverse components) for the TD case and the SD case.

Let’s first mention that, for the TD case, we only show fluctuations for \( T^* \geq 1.0 \): before that time, the fluctuations are negligible: they are below the threshold of the chosen contours. The TD results obviously show that the tranverse vorticity fluctuations are much larger than the longitudinal fluctuations. As we will see from the three-dimensional vorticity isosurfaces (Fig. 9), this can be explained by the fact that the instabilities generate “\( \Omega \)-loop”-like structures. The longitudinal vortex tubes are strongly bent and the resulting loops have transverse vorticity of opposite direction on each side of the loop; hence, the large transverse vorticity fluctuations. At \( T^* = 2.0 \), the transverse fluctuations in the outer vortex region are still at least twice larger than the longitudinal fluctuations. At that time, the largest fluctuations are observed in the primary vortex core. We note, however, that the fluctuations are high on an annulus around the vortex core; the core itself remains essentially laminar.

We now turn to the SD case. The first observation is that the fluctuation level is initially much higher than the TD case. Remember that, in the TD case, the initial perturbation level was set explicitly to a very low level. No explicit perturbation is added
in the SD case; however, the discrete procedure used to shed the wake vorticity at the inflow plane introduces a perturbation (at a still fairly low level). At $T^* = 1$, the fluctuation level in the SD is case is about 10 times higher than in the TD case. As will also be seen from the three-dimensional wake topology (Figs. 10 to 11), the wake also exhibits “Ω-loop” structures and the related vorticity fluctuations are observed; they appear however earlier than in the TD case. At $T^* = 1.5$, the region where large longitudinal fluctuations are observed appear to be much larger than in the TD case, even when compared with TD at $T^* = 2.0$. In comparison to the TD case, the SD case exhibits a more “homogeneous” turbulence: the order of magnitude of the fluctuations in the longitudinal and transverse directions are similar in the SD case. The large high longitudinal fluctuation region is further explained by the observed vortex “meandering” in the SD simulation: this can be seen in the three-dimensional wake visualizations (Figs. 10 to 15). Thus, in the averaged view, the primary vortex appears to be overly diffused. An instantaneous snapshot would reveal a turbulent, but still intense, vortex.

5.3 Axial flow component

One would be tempted to conclude from the vorticity fluctuation results that the essential differences between the time and the space developing flow configurations are related to the initial level of perturbation. While it is clear that a higher level of perturbation in the TD case would trigger the transition to turbulence earlier, there are other differences in the type of instabilities that cannot be explained solely by this effect.

The essential feature of the spatial developing flow, that was not discussed so far, is the presence of an axial flow component in the wake vortices. The fact that the roll-up of the vortex sheet occurs in a spiraling manner (instead of parallel vortex lines) introduces a significant tangential vorticity component in the rolled-up vortices. The axial flow introduces necessarily a velocity deficit in the rolled-up wake vortices. The result can be seen as the combination of a vortex and an axial jet. The time averaged axial velocity for the SD case is shown in Fig. 8. It can be seen that the axial velocity reaches values as low as $-4 b/T_0^*$.

The parameter that determines the type of instabilities that occur in a vortex-jet configuration is related to the swirl number, the ratio between the maximum axial velocity and the maximum tangential velocity: $S = u_{\theta,\text{max}}/u_{x,\text{max}}$. Based on the tangential velocity profiles from Fig. 21, $S = 1.8$ at $T^* = 0.5$ and $S = 0.87$ at $T^* = 1.0$. The low values of the swirl number indicate that the vortices have a strong jet component and will thus be more unstable than in the TD configuration (that has no axial jet).

The presence of the ground and the separation of a secondary vortex there further complicates the picture. At $T = 0.5$, it can be seen that the secondary vortex was formed together with a (less intense) positive axial jet. The swirl number for the secondary vortex is: $S = 2.3$ at $T^* = 0.5$ (based on the tangential velocity profile from Fig. 22).

We also note that, the strength of the axial velocity component is directly related to the lift coefficient: a “real” configuration (i.e., $C_L = 1.5$) will exhibit an axial velocity component about 4 times lower than the current configuration ($C_L = 6$). The swirl number
will be increased in the same proportion, thus reducing the gap between the temporal and spatial simulations.

5.4 Three-dimensional wake topology

Figure 9 shows the three-dimensional evolution of the vorticity field for the TD simulation: each frame shows two isosurfaces of vorticity norm, colored by the longitudinal vorticity component. At $T^* = 1$, short wave instabilities can be seen in the core region of the secondary vortex. As it interacts with the ground at $T^* = 1.25$, the instabilities strongly increase and, at $T^* = 1.5$, “Ω-loop”-like structures have formed and wrap themselves around the primary vortex. While vorticity continues to emanate from the boundary layer, it is continuously tilted and forms complicated loops that also wrap around the primary vortex. This corresponds to the phenomenon described in Section 5.2 about the transverse fluctuations in the TD case. By $T^* = 2.0$, the turbulence is fully developed (see also Fig. 25).

Figure 10 shows a snapshot, from three different viewpoints (top, side and perspective), of two isosurfaces of vorticity norm (same as for the TD case) when the SD case has reached its regime: one can see the global spiraling motion of the wake vortices. The three-dimensional dynamics are clearly quite different compared to the TD case. The secondary vortices interact with the primary vortex before having performed a complete revolution around it. The interaction is initiated by a medium wave instability that propagates on the secondary vortex. This is better illustrated by the time sequence shown in Fig. 11. One would even be tempted to described this instability on the secondary vortex as some kind of vortex “meandering”. Figures 12 and 13 give a closer look (top and side view) at the instability on the secondary vortex and its early interaction with the primary vortex (zoom on region $x/b = [4, 8]$ thus, $T^* = [0.5, 1]$). One can see how the medium wave instability periodically interacts with the primary vortex. It also shows that short wave instabilities with a double helical structure are present. Figures 14 and 15 consider the more distant wake (region $x/b = [8, 12]$ thus, $T^* = [1, 1.5]$). It shows how the secondary vortex partially reconnects with the primary through a very complex spiraling motion. Due to this interaction, the primary vortex appears to “burst” into two or even three helical vortices which then further “break down” into small-scale turbulence.

5.5 Time evolution of diagnostics

Let us now look into the time evolution of some diagnostics characterizing the wake dynamics up IGE for the three cases investigated.

We first define the center of a vortex. It is obtained by fitting a vorticity profile model (here a low order algebraic function) on the averaged vorticity field in a least mean square sense. The vortex center is then defined as the center of the fitted function. This allows to obtain a good definition which is insensitive to local fluctuations in the averaged field when the flow has become turbulent. When the flow is still laminar this essentially comes down to defining to vorticity center based on the local extremum of vorticity (as is usually done).
Figure 16 shows the trajectories of the primary vortex center in the right-half plane for the three investigated cases. The initial trajectories of the 2D and TD cases are identical in the early roll-up phase. The SD trajectory is slightly lower, probably due to the downwash velocity induced by the lifting line itself in the near wake region. During the rebound, the TD trajectory is somewhat shifted inwards (compared to the 2D case): this is explained by the earlier separation of the boundary layer (as already mentioned in Section 5.1). The rebound loop in the SD case is completed earlier compared to the 2D and TD cases. However, as the SD case becomes turbulent immediately afterwards, the motion of the primary vortex is “decelerated” and, at $T^* = 1.5$, the primary vortex center locations are very close for all cases. The 2D and TD trajectories are shown up to $T^* = 2.0$, by that time the 2D case has clearly separated from the TD trajectory due to the absence of three-dimensional effects.

Based on the vorticity center, we can define the vortex circulation profile. It is determined using

$$\Gamma(r) = \int_0^{2\pi} \int_0^r \omega_x(r', \theta) r' dr' d\theta$$

(16)

where the integration radius is taken from the vortex center, and the vorticity is respectively the instantaneous, the space averaged and the time averaged vorticity field for the 2D, TD and SD cases. Based on the circulation profile, one usually defines the $\Gamma_{5-15}$: the average circulation contained between 5 and 15 meter for a wake generated by a 60 meter wing span (thus between $1/12b$ and $1/4b$). In our case however, this quantity does not have much sense as the initial height of the wake above the ground is $b/4$ (the $\Gamma_{5-15}$ quantity immediately includes opposite sign vorticity from the boundary layer and the secondary vortex). Therefore, we here define $\Gamma_{\text{max}}$: the maximum of the circulation profile. This quantity is a good indicator of the total circulation contained in the primary vortex. Figure 17 shows the “maximum” circulation, $\Gamma_{\text{max}}$, for the primary vortex as a function of time. The evolution of $\Gamma_{\text{max}}$ for the 2D and TD cases are essentially identical up to $T^* = 1.2$, as the TD case remains two-dimensional until then. Then, when the TD flow becomes three-dimensional at $T^* = 1.4$, the circulation decreases rapidly. The SD case is only slightly different up to $T^* = 0.8$; past that time, the rapid circulation decrease occurs. In terms of the spatial location in the wake, this corresponds to $x/b = 6.4$: referring to Figs. 12 and 13, one can see that this corresponds to the first location where the secondary vortex periodically interacts with the primary vortex.

The circulation profile, $\Gamma(r)$, can also be used to evaluate the vortex induced tangential velocity profile,

$$u_\theta(r) = \Gamma(r)/(2\pi r).$$

(17)

The maximum tangential velocity, $u_{\theta, \text{max}}$, is determined from this profile, along with the “effective vortex core radius”, $r_c$: the radius where the maximum occurs. Figures 18 and 19 show the temporal evolution of those quantities. Interestingly, $u_{\theta, \text{max}}$ and $r_c$ for the TD case remain close to the 2D case for a longer time than it was the case for $\Gamma_{\text{max}}$. This might indicate that, while the total circulation of the primary vortex decreases due to three-dimensional effects and turbulence, the inner vortex core remains unaffected (at least for some time) indicating a laminar vortex core. The picture for the SD case is dramatically different: at $T^* = 0.8$ (here perfectly in sync with the decrease of $\Gamma_{\text{max}}$), $u_{\theta, \text{max}}$ decreases rapidly while $r_c$ increases accordingly. At $T^* = 1.5$, $u_{\theta, \text{max}}$ is about 2.5
times lower than 2D and TD cases and $r_c$ is about twice larger. This indicates that the primary vortex is very rapidly destroyed in the SD case: at $T^* = 1.0$, the vortex core is already strongly “diffused” according to this picture. While, the additional instabilities due to the presence of the velocity deficit certainly play an important role here, we think that this effect is combined with another phenomenon: some kind of vortex meandering that induces oscillatory motions on the vortex center. The average vortex core therefore looks more diffused than it really is.

Figures 20 and 21 further illustrate the previous comments by showing respectively the circulation profiles, $\Gamma(r)$, and the tangential velocity profiles, $u_\theta(r)$, at different times for all cases. Figure 22 shows $\Gamma(r)$ and $u_\theta(r)$ at $T^* = 0.5$ for the secondary vortex in the SD case: these results (with the axial velocity results in Fig. 8) can be used to determine the swirl number for the secondary vortex at that time.

Another interesting view of the vortex system is obtained when considering the energy of the averaged velocity fields (either longitudinally space averaged in the TD case or time averaged in the SD case). The time evolution of the cross-plane kinetic energy (i.e., only the transverse components of the velocity) is shown in Fig. 23. The results are very much in line with the previous observation. It is important to notice that the slope of the rapid energy decay region for the SD case is much steeper than the TD case. Again, these results show that the SD case transitions to turbulence earlier but also that the energy decay mechanism is significantly enhanced.

Finally, Figs. 24 and 25 presents a detailed view of the energy evolution for the TD case. In particular, Fig. 25 shows the modal energy spectrum at different times. At $T^* = 1$, the spectrum shows that there is a competition between medium wave instabilities and short wave instabilities. Later, the flow becomes fully turbulent and the spectrum exhibits the typical large- to small-scale energy cascade.

6 Conclusion

We have presented all the simulation results for the three investigated configurations of “one case”: the roll-up and evolution of the wake generated by an elliptical wing IGE. The three configurations differ by the number of physical features they capture. The simplest configuration is that of a purely 2-D simulation IGE: its use is clearly limited, but it allowed to have an accurate DNS reference. The three-dimensional longitudinally uniform configuration is able to account for the three-dimensional instabilities and the turbulent evolution of the wake, but it makes the hypothesis of a globally parallel vortex wake. In order to take into account the fact that the vortex sheet actually rolls up in a spiralling manner, it is necessary to model the wake as it is produced in reality: in a spatially developing (SD) configuration. This was done in the third configuration. The objective of this study was to investigate the effects related to the longitudinally developing character of the wake by comparing the results from the different configurations.

The following observations were made:

- During the initial phase, when the flow is essentially two-dimensional, the three-dimensional results (TD and SD cases) match very well with the 2D reference case.
The SD case shows greater differences than the TD case but this is logical as additional three-dimensional effects are present, even before three-dimensional instabilities appear.

- All the simulation results show that the SD case transitions earlier to turbulence than the TD case. This can be partly related to the fact that the initial perturbation level in the TD case is lower than in the SD case.

- The growth rate of the instabilities in the SD case also appears to be much larger than for the TD case. One important feature, that is not present in the TD simulation, is the fact that the spatially developing wake induces an axial velocity defect in the primary vortices. The resulting wake vortices can thus be seen as a combined vortex-jet configuration, which exhibits additional instabilities. The strength of the velocity defect is furthermore directly related to the wing lift coefficient, which was set to a large value \((C_L = 6)\) due to computational resources limitations. In a more realistic configuration, the axial velocity component would thus be lower, which in turn would reduce the strong instabilities, bringing the SD results closer to the TD results.

One way to investigate if the differences between the TD and SD configurations are related to the axial velocity component would be to investigate a longitudinally uniform configuration with a comparable axial “jet” superposed on the initial vortex. This configuration would certainly be closer to the space developing configuration. It would however not take into account features such as the positive axial velocity that appears in the secondary vortices and, most importantly, the fact that the vortex lines are not parallel in reality. From the SD simulation, we suspect that the observed meandering motion of the primary and secondary vortices might be related to the fact that the vortices describe a spiralling motion.

To conclude, we think the present results show that longitudinally uniform wake simulations (TD) are certainly a valid tool to investigate wake vortices. While our results showed sometimes large differences between the space and time developing configurations, one must remember that the investigated space developing case is somewhat extreme (due to its close distance to the ground and its very high lift coefficient). Furthermore, one could improve the TD simulations by introducing some axial flow. However, it also appears that some features that can only be taken into account in a spatially developing configuration have an important effect.

The next step would therefore be to perform a space developing simulation in a more realistic configuration (i.e., a lower lift coefficient). This would greatly increase the computational demands. The VIC-PFM code, in its present state, would be able to run such a simulation: it is a matter of running a four times larger domain using four times more processors. It is also a matter of handling four times larger files and of analyzing the results: something quite demanding! ... yet doable.

Finally, it should be noted that the space developing configuration is still very different from real aircraft wakes as we here consider a “tracted wing” rather than a self-propulsive configuration. In reality, beside the velocity defects observed here, velocity excess regions exist (from the jets) as the total jet impulse has to compensate the induced and profile
drag. In this case, where jets and wakes interact with vortices, there are many more possibilities for interaction between streamwise and transverse vorticity and associated instabilities.
Figure 3: 2-D Time developing DNS. Vorticity contours are shown for the right half plane at distinctive times: $T^* = -t^*$ and $\Omega = \omega T^*_0$. 
3-D Time developing

Figure 4: 3-D Time developing LES. Space averaged vorticity contours are shown for the right half plane at distinctive times: $T^* = \frac{t}{\tau_d}$ and $\Omega = \omega T_0^*$. 
Figure 5: 3-D Space developing LES. Time averaged vorticity contours are shown for the right half plane in distinctive slices: $T^* = \frac{x}{U_\infty} \frac{1}{T_0}$ and $\Omega = \omega T_0$. 
Figure 6: 3-D Time developing LES. Space averaged fluctuations of longitudinal vorticity, \( \sqrt{\omega_x'^2} \), (left) and of transverse vorticity, \( \sqrt{\omega_y'^2 + \omega_z'^2} \), (right) at distinctive times: \( T^* = T/T_0 \) and \( \Omega' = \omega' T_0^* \). Two positive (plain) and negative (dashed) contours of averaged longitudinal vorticity are also shown.
3-D Space developing

Figure 7: 3-D Space developing LES. Time averaged fluctuations of longitudinal vorticity, $\sqrt{\omega_x^2}$, (left) and of transverse vorticity, $\sqrt{\omega_y^2 + \omega_z^2}$, (right) in distinctive slices: $T^* = \frac{x}{U_\infty T_0}$ and $\Omega' = \omega' T_0$. Two positive (plain) and negative (dashed) contours of averaged longitudinal vorticity are also shown.
Figure 7: (cont.) 3-D Space developing LES. Time averaged fluctuations of longitudinal vorticity, $\sqrt{\omega_x'^2}$, (left) and of transverse vorticity, $\sqrt{\omega_y'^2 + \omega_z'^2}$, (right) in distinctive slices: $T^* = \frac{t}{u_\infty}$ and $\Omega' = \omega'/T_0$. Two positive (plain) and negative (dashed) contours of averaged longitudinal vorticity are also shown.
Figure 8: 3-D Space developing LES. Time averaged axial velocity contours are shown for the right half plane in distinctive slices: $T^* = \frac{1}{T_0} \sqrt{\frac{U_x}{U_\infty}}$ and $U_x = u_x \frac{1}{T_0}$. 
3-D Time developing

\[ T^* = 1.00 \]

\[ T^* = 1.25 \]

**Figure 9:** 3-D Time developing LES. Isosurfaces of vorticity norm, \( ||\Omega|| = 80 \) (opaque) and 25 (translucent), colored by the longitudinal component of vorticity at distinctive times.
3-D Time developing

$T^* = 1.50$

$T^* = 1.75$

Figure 9: (cont.) 3-D Time developing LES. Isosurfaces of vorticity norm, $||\Omega|| = 80$ (opaque) and 25 (translucent), colored by the longitudinal component of vorticity at distinctive times.
Figure 9: (cont.) 3-D Time developing LES. Isosurfaces of vorticity norm, $||\Omega|| = 80$ (opaque) and 25 (translucent), colored by the longitudinal component of vorticity at distinctive times.
Figure 10: 3-D Space developing LES. Isosurfaces of vorticity norm, $||\Omega|| = 80$ (opaque) and 25 (translucent), colored by the longitudinal component of vorticity when flow has reached its regime state. Top and side views. The locations of the time averaging planes used in Figs. 5, 7 and 8 are also shown.
Figure 10: (cont.) 3-D Space developing LES. Isosurfaces of vorticity norm, $||\Omega|| = 80$ (opaque) and 25 (translucent), colored by the longitudinal component of vorticity when flow has reached its regime state. Perspective view.
Figure 11: 3-D Space developing LES. Time sequence of isosurfaces of vorticity norm, $||\Omega|| = 80$ (opaque) and 25 (translucent), colored by the longitudinal component of vorticity. Top view of right half-domain for each time frame are shown. $t = 0$ here corresponds to an arbitrary time (the same as in Fig. 10), once the system has reached its regime.
Figure 12: 3-D Space developing LES. Time sequence of isosurfaces of vorticity norm, $||\Omega|| = 80$ (opaque) and 25 (translucent), colored by the longitudinal component of vorticity. Top view of right half-domain (zoom on region $x/b=[4,8]$) for each time frame are shown. $t = 0$ here corresponds to an arbitrary time (the same as in Fig. 10), once the system has reached its regime.
Figure 13: 3-D Space developing LES. Time sequence of isosurfaces of vorticity norm, $||\Omega|| = 80$ (opaque) and 25 (translucent), colored by the longitudinal component of vorticity. Side view of right half-domain (zoom on region $x/b=[4,8]$) for each time frame are shown. $t = 0$ here corresponds to an arbitrary time (the same as in Fig. 10), once the system has reached its regime.
Figure 14: 3-D Space developing LES. Time sequence of isosurfaces of vorticity norm, $||\Omega|| = 80$, colored by the longitudinal component of vorticity. Top view of right half-domain (zoom on region $x/b=[8, 12]$) for each time frame are shown. $t = 0$ here corresponds to an arbitrary time (the same as in Fig. 10), once the system has reached its regime.
Figure 15: 3-D Space developing LES. Time sequence of isosurfaces of vorticity norm, $||\Omega|| = 80$, colored by the longitudinal component of vorticity. Side view of right half-domain (zoom on region $x/b=[8,12]$) for each time frame are shown. $t = 0$ here corresponds to an arbitrary time (the same as in Fig. 10), once the system has reached its regime.
Figure 16: Trajectories of primary vortex locations for the 2-D DNS (2D), the 3-D time developing LES (TD) and the 3-D space developing LES (SD). Symbols are shown at constant time intervals: small symbols for $\Delta T^* = 0.1$, large symbols for $\Delta T^* = 0.5$.

Figure 17: Time evolution of maximum circulation, $\Gamma_{\text{max}}$, for the 2-D DNS (2D), the 3-D time developing LES (TD) and the 3-D space developing LES (SD).
Figure 18: Time evolution of $u_{\theta, \text{max}}$ for the 2-D DNS (2D), the 3-D time developing LES (TD) and the 3-D space developing LES (SD).

Figure 19: Time evolution of $r_c$ for the 2-D DNS (2D), the 3-D time developing LES (TD) and the 3-D space developing LES (SD).
Figure 20: Circulation profiles, $\Gamma(r)$, for the 2-D DNS (plain), the 3-D time developing LES (dash-dot) and the 3-D space developing LES (dash).
Figure 21: Tangential velocity profiles, $u_\theta(r)$, for the 2-D DNS (plain), the 3-D time developing LES (dash-dot) and the 3-D space developing LES (dash).
Figure 22: 3-D Space developing LES. Circulation profile, $\Gamma(r)$ (left), and tangential velocity profile, $u_\theta(r)$ (right), for the secondary vortex at $T^* = \frac{x}{U_\infty} \frac{r_0}{T_0} = 0.5$.

Figure 23: Time evolution of cross-plane kinetic energy for the 2-D DNS (2D), the space averaged 3-D time developing LES (TD) and the time averaged 3-D space developing LES (SD).
Figure 24: 3-D Time developing LES. Time evolution of kinetic energy for the space averaged field (TD 2D) and for the complete 3-D field (TD 3D).

Figure 25: 3-D Time developing LES. Modal energy spectra for distinctive times.
References


