Elliptic instability in a pair of vortices with axial jet

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Visualisations of aircraft trailing wakes

- known to be dangerous for a following aircraft
- minimum separation distances → limits airport capacity
Vortex system in the wake of a civil aircraft
– typical take-off/landing configuration –

wing tip vortex
flap tip vortex
merged vortex
co-rotating vortices
merging
counter-rotating vortices
Vortices **WITH** axial core flow

Vortex pairs

- **co-rotating**
- **counter-rotating**
Overview

✓ Introduction

➢ Reminder on the elliptic instability
  • Numerical study
  • Experimental results
  • Conclusion / outlook
Elliptic instability mechanism

Perturbations of a vortex (Kelvin modes)
\[ u(r) \cdot \exp[i(kz + m\Theta - \omega t)] \]

Resonance between two modes \((m_1, k_1, \omega_1)\) and \((m_2, k_2, \omega_2)\) and the induced strain is possible if
\[
|m_1 - m_2| = 2 \\
k_1 - k_2 = 0 \\
|\omega_1 - \omega_2| = 0
\]

Amplification of perturbations
Instability
Elliptic instability **without** axial flow

Counter-rotating vortices without axial flow  
Leweke & Williamson (1998)

Co-rotating vortices without axial flow  
Meunier & Leweke (2002)

sinuous deformation with a short wavelength
Theoretical prediction for a Gaussian vortex in a stationary strain field
- Lacaze & Le Dizès 2005 –

Positive growth rate colour-coded
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Flow parameters

Base flow is generated and **frozen**
Initial condition: **gaussian** vortices.

\[ Re = \frac{\Gamma}{\nu} : \text{Reynolds number} \]
\[ a/b : \text{rescaled core size} \]
\[ W_0 = \frac{2\pi a W(r=0)}{\Gamma} : \text{inverse of Swirl number} \]

Fourier expansion on the axial dimension for one wavelength \( \lambda \)
\( \Rightarrow \) **linear stability analysis** of a 2D base flow relatively to 3D perturbations

\[ k = \frac{2\pi a}{\lambda} : \text{axial wave number} \]
Co-rotating vortices, $a/b=0.14 \, \text{Re}=14000$

similarities with the counter-rotating case: **Lacaze & Le Dizès 2005**
Characterisation of the modes. Example: mode 5

- frequency $|\omega|$  
- azimuthal wavenumber $|m|$  
- label of the normal modes $n$

$vorticity perturbation field$

$|m|=0$
$|m|=2$

recomposed vorticity field
Characterisation of the modes. Example: mode 5

Time dependence

- $u(r) \cdot \exp[i(kz + m\theta - \omega t)]$
- $k > 0$ (imposed) $\Rightarrow m < 0$
  $\Rightarrow \omega < 0$

$(m_1, m_2) = (0, -2)$

Spatial dependence
\((m_1, m_2) = (-1, 1)\)

\((n_1, n_2) = (1, 2)\)
\( (m_1, m_2) = (0, -2) \)

\( (n_1, n_2) = (3, 2) \)

\( (n_1, n_2) = (1, 2) \)

\( n=1 \)

\( n=2 \)

\( n=3 \)
\[(m_1, m_2)=(-1, -3)\]
\((m_1, m_2)=(-2, -4)\)
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➢ Experimental results

• Conclusion / outlook
Facilities and setup

**Water channel:**
- Free surface
- Test section 37cm × 50cm × 150cm
- Free stream velocity: \( U_o = 57.5 \text{ cm/s} \)

**Wings:**
- NACA 0012 profiles
- Chord \( c = 100 \text{ mm} \)
- Span: 160 mm and 300 mm
- \( Re_c = U_o c / \nu = 5.7 \times 10^4 \)
- \( \alpha = 7^\circ \)

**Methods:**
- Dye visualisations
- Stereo-PIV velocity measurements
Counter-rotating vortices

- $Re_c = 57,000$
- $\alpha = 7^\circ$

Stereo-PIV
Development of the instability
Visualisation
Characterisation of the mean field
– Stereo-PIV –

Vorticity (s⁻¹)

Axial velocity (cm/s)

Instantaneous stereo-PIV fields at P1
Flow characterisation

- Each vortex is recentered
- 300 fields are averaged
- **Gaussian** vortex parameters are extracted

\[ Re=15800, \ a/b=0.16 \ et \ W_0=0.44 \]
Wave length and wave number

Periodic perturbation: azimuthal wave number $|m| = 2$
average wave length $\lambda \approx 1.7$ cm, $ka \approx 2.05 \pm 0.14$
Measurement of the perturbation amplitude $A$ for 5 different $t^*$
Estimation of the growth rate

$$2\pi b^2 \sigma / \Gamma = 0.71 \pm 0.08$$
Comparison with numerical results

\((-2,0,1)\) : most unstable mode for
\(W_0=0.44, \; a/b=0.165\) et \(Re=16800\)

Total vorticity

Linear stability analysis of a gaussian vortex pair
Summary – Conclusions

Numerical and experimental study of the elliptic instability in two vortices with axial core flow at high Reynolds number

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Theoretical predictions Lacaze et al.

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Phenomenon well understood and predictable

See Schaeffer's presentation for the consequences on merging and far wake